REVIEW OF THERMODYNAMICS

by

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THERMODYNAMICS

Outline

1. Ideal Gas Laws
2. 1st Law - Closed System
3. 1st Law - Open System
4. Steam Tables
5. Cycles
   - Carnot
   - Reversed Carnot
   - Otto
   - Rankine
   - Reversed Rankine
6. Psychrometric Chart
7. Psychrometric Relations
8. Partial pressures
9. Heat transfer

REVIEW OF THERMODYNAMICS

Ideal Gas Laws (p. 43)

$$p v = RT \qquad \text{where } R = \frac{\bar{R}}{\text{Mol. Wt}}$$

$$\bar{R} = 8.314 \frac{kJ}{kmole\,K} \text{ or } 1545 \frac{ft\text{-}lb_f}{lbmole\,{}^{\circ}R}$$

Note: Pages 1 & 2 of Reference Manual provide constants and conversion factors. P. 52 in reference manual provides Molecular Wt and other data for common fluids.

Another version of Ideal Gas Laws

$$pV = mRT \qquad (V = \text{Volume}, \; m = \text{mass})$$

and

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \quad \text{or} \quad \frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

Note: Remember to convert Temperatures to K or °R in ideal gas law equations.

For ideal gases and constant specific heats,

$$\Delta h = c_p \Delta T$$

$$\Delta u = c_v \Delta T$$

and $$\Delta s = c_p \ln(T_2/T_1) - R \ln(P_2/P_1)$$

$$\Delta s = c_v \ln(T_2/T_1) + R \ln(v_2/v_1)$$

If entropy is constant,

$$p_1 v_1^{k} = p_2 v_2^{k} \qquad k = \frac{c_p}{c_v}$$

$$T_1 p_1^{\frac{(1-k)}{k}} = T_2 p_2^{\left(\frac{1-k}{k}\right)}$$

EXAMPLE #1 A rigid container of $0.5 m^3$ contains carbon dioxide at 5 kPa pressure and 17 °C. Calculate the mass of carbon dioxide in the container.

SOLUTION:

$V = 0.5 m^3$
CO_2
$T = 17°C$
$P = 5 kPa$

Assume ideal gas conditions.

$T = 273°C + 17°C = 290 K$

Mol. Wt = 44

$$PV = mRT$$

$$m = \frac{PV}{RT} = \frac{(5 kPa)(0.5 m^3)}{\left(\frac{8.314 \frac{kJ}{kmole\,K}}{\frac{44 kg}{kmole}}\right)(290 K)} = \boxed{0.046 kg}$$

EXAMPLE #2 If the gas in the pressure vessel in EXAMPLE #1 is now heated to 117°C, calculate:

(a) New pressure
(b) Change in entropy in $kJ/(kg\,K)$

SOLUTION: Since $V = \text{Constant}$ and $\frac{p_1 \cancel{V_1}}{T_1} = \frac{p_2 \cancel{V_2}}{T_2}$

$$p_2 = p_1\left(\frac{T_2}{T_1}\right) \qquad T_2 = 273 + 117 = 390\,K$$

$$p_2 = 5\,kPa\left(\frac{390}{290}\right) \qquad T_1 = 290\,K$$

$$\boxed{p_2 = 6.7\,kPa} \quad (a)$$

Change in entropy $(s_2 - s_1)$ can be found from $\Delta s = c_p \ln(T_2/T_1) - R \ln(p_2/p_1)$

or

$$\Delta s = c_v \ln(T_2/T_1) + R \ln(\cancel{v_2/v_1})^{\to 0}$$

Const. volume

$c_v = 0.652 \frac{kJ}{kg\,K}$
p. 52

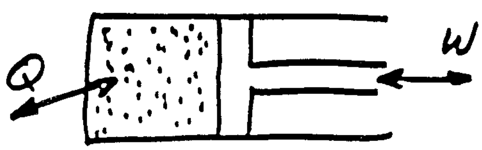
$$\Delta s = 0.657 \ln\left(\frac{390}{290}\right) = 0.195 \frac{kJ}{kg \cdot K}$$

or $s_2 - s_1 \approx \boxed{0.20 \frac{kJ}{kg\,K}}$ (b)

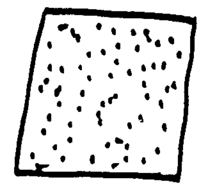
CLOSED THERMODYNAMIC SYSTEMS – no mass crosses the boundaries.

Two Common Closed Systems

Piston-cylinder



Rigid Container



const. volume as well as const. mass

1st Law

$$Q - W = \Delta U$$

Sign Convention ⇒ Heat Added is positive.
Work done by system is positive.

OPEN THERMODYNAMIC SYSTEMS (Control Volumes)
Mass and energy cross boundaries.

p.44

1st Law (Steady state conditions)

$$\sum_i \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i\right) - \sum_e \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e\right) + \dot{Q}_{in} - \dot{W}_{out} = 0$$

Conservation of Mass Eq.

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e$$

Examples of Open System Components: turbines, compressors, pumps, throttling devices, heat exchangers

Closed System Example

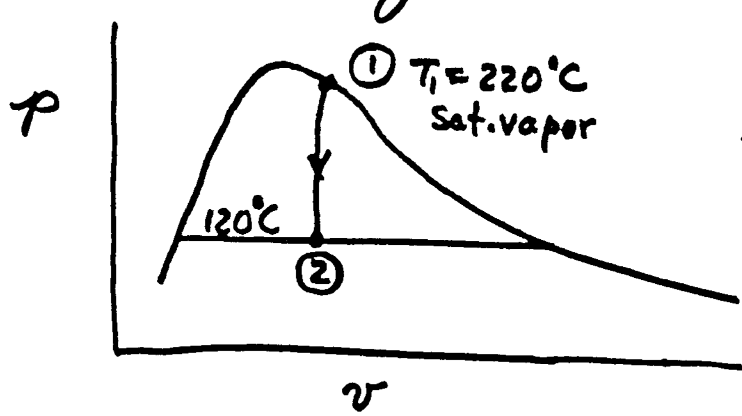
EXAMPLE #3 A rigid pressure vessel contains steam at 220°C, saturated vapor. Heat is removed from the system until the final temperature is 120°C. The heat transfer for this process is most nearly:

A. −2088 kJ/kg
B. −1905 kJ/kg
C. 1905 kJ/kg
D. 2088 kJ/kg

SOLUTION:

1st Law $Q - W = \Delta U$ or $q - w = \Delta u$, on a unit mass basis

The vessel is rigid, no work is done, so first law reduces to $q = \Delta u = u_2 - u_1$



$v_1 = v_2$. At 220°C & sat. vapor, find $v_1 = v_2 = 0.08619 \frac{m^3}{kg}$ & $u_1 = 2602.4 \frac{kJ}{kg}$

Point 2 lies in the mixture region (under the dome). To find u_2, find x (quality).

From p. 43 $v = x\, v_{fg} + v_f$

$$x = \frac{v - v_f}{v_{fg}}$$

From steam tables, p.48, at 120 °C

$$\left.\begin{aligned} v_f &= 0.001060 \ m^3/kg \\ v_g &= 0.8919 \ m^3/kg \\ u_f &= 503.5 \ kJ/kg \\ u_{fg} &= 2025.8 \ kJ/kg \\ u_g &= 2529.3 \ kJ/kg \end{aligned}\right\} \text{ at State 2 } = 120\,°C$$

$$x = \frac{0.08619 - 0.00106}{0.8919 - 0.00106} = 0.0956$$

$$u_2 = x\,u_{fg} + u_f$$

$$u_2 = 0.0956(2025.8) + 503.5$$

$$u_2 = 697 \ kJ/kg$$

Key is B

$$q = \Delta u = u_2 - u_1 = 697 - 2602.4 = \boxed{-1905 \ \frac{kJ}{kg}}$$

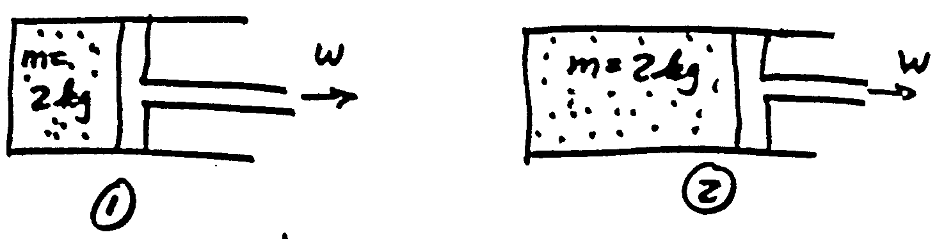
The problem was set up like the problems on the exam. In analyzing the problem, recognize that heat is removed from the system; therefore q must be negative. We can eliminate answers C and D immediately, since they are positive.

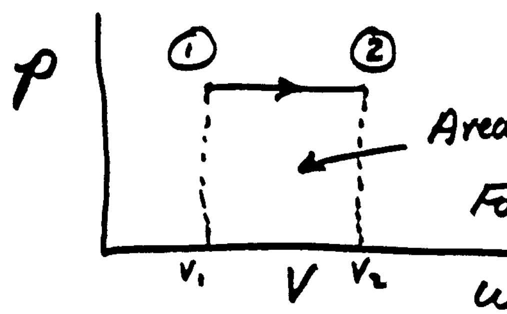
Let's look at another example of a closed system problem.

EXAMPLE # 4 Two Kilograms of air at 800 kPa and $T = 127°C$ are expanded at constant pressure to a temperature of 227°C. Assume ideal gas conditions and constant specific heats. The heat transferred during the process is most nearly:

A. 72 kJ
B. 100 kJ
C. 144 kJ
D. 200 kJ

SOLUTION:





Area = Work = $\int p\,dV$

For p = Const.

$w = p\Delta V = p(V_2 - V_1)$

1st Law

$q = \Delta u + w$

$Q = m(u_2 - u_1) + p(V_2 - V_1)$

In the analysis of the problem, there is a "short" approach and a "long" approach. The "long" way is to calculate u_1, u_2, V_1, and V_2, add them together and solve for Q. The "short" way is to recognize that the definition of enthalpy, h, is $u + pv$, or $H = U + pV$. Further, for ideal gases, $h = c_p(T)$.

$\therefore$ Since $u_2 + pv_2 = h_2$ and $u_1 + pv_1 = h_1$
and $h = c_p T$ (p. 43)

$Q = m\, c_p (T_2 - T_1)$ $\quad c_{p\,air} = 1\ kJ/(kg \cdot K)$
p. 52 Table

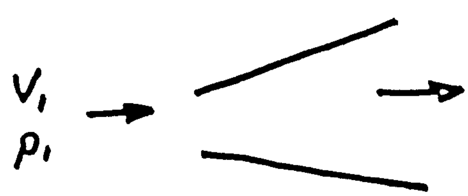
$$Q = (2\,kg)\left(1\,\frac{kJ}{kg\,K}\right)(500 - 400)$$

$$\boxed{Q = 200\,kJ}$$ $\therefore$ key is D

Note: Since we are taking the difference between temperatures, we did not need to convert to absolute units of Kelvin. However, I usually convert to Kelvin (or Rankine, in English units) automatically in ideal gas problems.

Now let's look at some open system or control volume problems. We'll rewrite the first law as a rate equation for steady state conditions. (For now, we will assume only one inlet and exit condition.) p. 44

$$\dot{m}_1 \left(h_1 + \frac{V_1^2}{2} + g z_1\right) - \dot{m}_2 \left(h_2 + \frac{V_2^2}{2} + g z_2\right) + \dot{Q}_{in} - \dot{W}_{out} = 0$$



Diffuser

$V_1 > V_2$

$P_2 > P_1$

Nozzle

$V_2 > V_1$

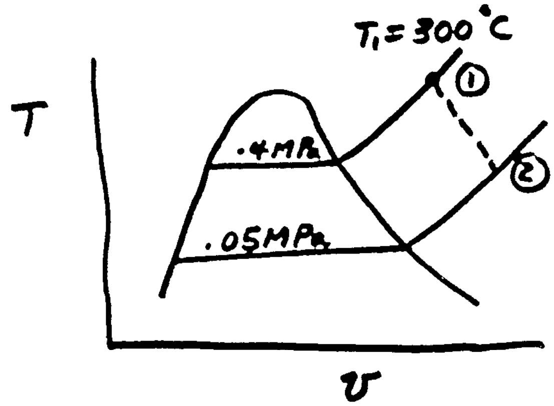
$P_2 < P_1$

Typical Assumptions
$\Delta z = 0$
$\dot{W} = 0$
$\dot{Q} = 0$

EXAMPLE #5:

Steam flows through a nozzle operating at steady state conditions. $P_1 = 0.40\,MPa$ & $T_1 = 300°C$. At the nozzle exit, $P_2 = 0.05\,MPa$, and the velocity, V_2, is $300\,m/s$. Ignoring the entering K.E., calculate the final enthalpy.

SOLUTION:



1st Law $\quad \dot{m}_1 \left(h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{g z_1}\right) - \dot{m}_2 \left(h_2 + \frac{V_2^2}{2} + \cancel{g z_2}\right) + \cancel{\dot{Q}_{in}}^{\,0} - \cancel{\dot{W}_{out}}^{\,0} = 0$

neglect in this problem   neglect   neglect

$$\cancel{\dot{m}_1} h_1 - \cancel{\dot{m}_2}\left(h_2 + \frac{V_2^2}{2}\right) = 0$$

$$h_2 = h_1 - \frac{V_2^2}{2}$$

$$h_2 = 3066.8 - \left(\frac{300^2}{2}\right)\left(\frac{m^2}{s^2}\right)\left(\frac{1\,N}{1\,kg \cdot \frac{m}{s^2}}\right)\left(\frac{1\,kJ}{10^3\,N \cdot m}\right)$$

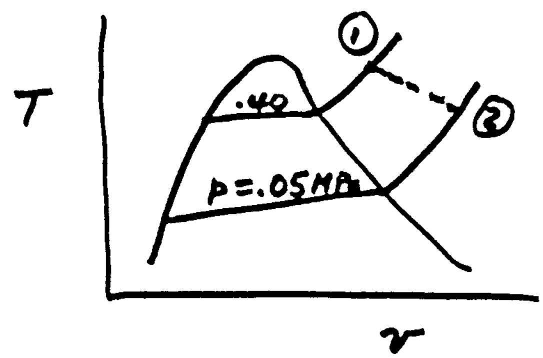
$$h_2 = 3066.8 - 45$$

$$\boxed{h_2 = 3021.8\ kJ/kg}$$

From p. 49
At $P_1 = .4\,MPa$ & $300°C$
$h_1 = 3066.8\ \frac{kJ}{kg}$

EXAMPLE #6 SAME PROBLEM AS IN EXAMPLE #5, except ask for the temperature of the steam leaving the nozzle.

SOLUTION



This is a table lookup and interpolation problem. We calculate h_2 as before, i.e.,

$$h_2 = h_1 - \frac{V_2^2}{2} = 3021.8 \text{ kJ/kg}$$

Now go to page 49 in Reference Manual and locate the point of intersection of

$$p_2 = 0.05 \text{ MPa} \text{ \& } h_2 = 3021.8 \text{ kJ/kg}$$

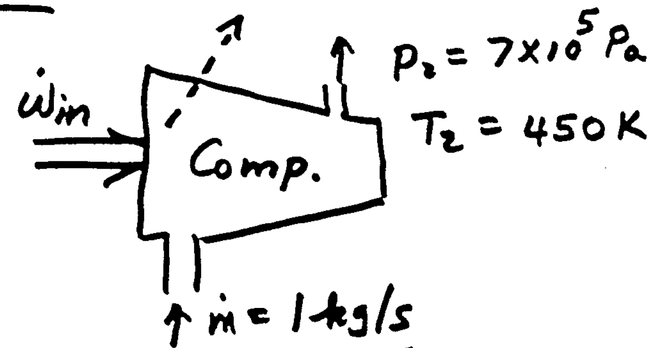
We see that the temperature of State 2 is between 250°C (h = 2976.0) and 300°C (h = 3075.5)

Using linear interpolation, $\boxed{T_2 = 273°C}$

EXAMPLE # 7

One Kilogram per sec of nitrogen gas enters a compressor operating at steady state. The entering conditions are $p_1 = 1\times10^5\ Pa$, $T_1 = 290K$, and the exit conditions are $p_2 = 7\times10^5\ Pa$, $T_2 = 450K$. Heat losses to the surroundings occur at a rate of 200 kJ/min. Neglect potential and kinetic energy changes, assume ideal gases, and calculate the work done in KW.

SOLUTION:



$$\dot{m}h_1 - \dot{m}h_2 + \dot{Q}_{in} - \dot{W}_{out} = 0$$

For ideal gases $\quad h = c_p(T)$

For Nitrogen
$C_p = 1.04 \frac{kJ}{kg\,K}$
p. 52

$$\dot{W}_{comp} = \dot{m}c_pT_1 - \dot{m}c_pT_2 + \dot{Q}_{in}$$

$$\dot{W}_{comp} = \left(1\frac{kg}{s}\right)\left(1.04\frac{kJ}{kg\cdot K}\right)(290) - (1)(1.04)(450) - 200\frac{kJ}{min}\times\frac{min}{60s}$$

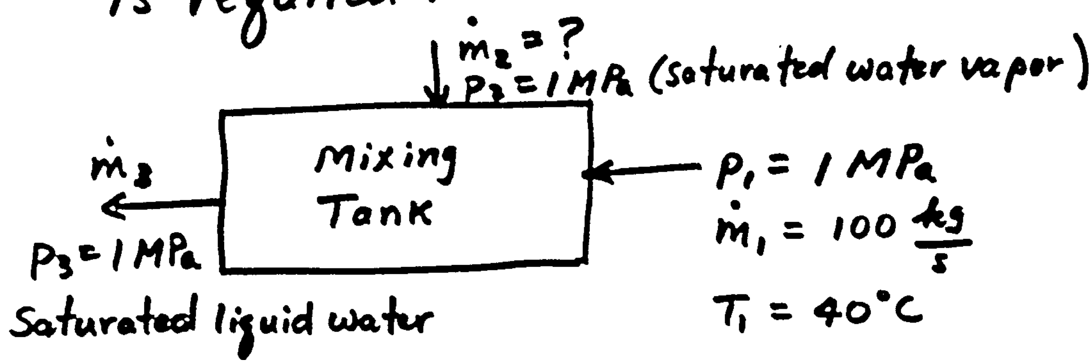
$$\dot{W}_{comp} = (301.6 - 468 - 3.33)\frac{kJ}{s}$$

$$\boxed{\dot{W}_{comp} = -169.7\ KW}$$

Note: 1 watt = $1\,J/s$; $1\,kW = 1\,\frac{kJ}{s}$

Also note that $\dot{Q}_{in}$ is negative because it's heat lost (By our sign convention), and $\dot{W}_{comp}$ is negative because it is a work input to our system.

EXAMPLE #8: (Mixing device). Steam is mixed with a subcooled liquid to raise the liquid temperature to saturated conditions. A mixing tank is used. For the conditions shown, how much steam is required?



SOLUTION: From Conservation of mass equation, $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$. Assume no losses in heat transfer, and neglect ΔKE and ΔPE. No Work is done.

$$\therefore\ \dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

Now we need to find the enthalpies from the steam tables.

State 1 – Liquid is subcooled (i.e., a compressed liquid) and enthalpy is found from the <u>temperature</u> of the liquid at saturation conditions.

From p. 48 & $T = 40^\circ C$, $\underline{h_1 = 167.57 \frac{kJ}{kg}}$

State 2 – Steam is saturated vapor at 1 MPa.

Again on p. 48, the pressure closest to 1 MPa is 1.0021 MPa, so we will use that pressure.

$\underline{h_2 = 2778.2 \ kJ/kg}$

State 3 – Liquid water leaves the heat exch. at saturated liquid conditions.

Again at 1.0021 MPa (Closest value to 1 MPa)

$\underline{h_3 = 763.22 \ kJ/kg}$

Using our two equations, $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$ and $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$

$$(100 \ kg/s)(167.57)\frac{kJ}{kg} + \dot{m}_2 (2778.2) = (100 + \dot{m}_2) 763.22$$

$$\boxed{\dot{m}_2 = 29.6 \ kg/s}$$

CYCLES

There are five cycles you should be familiar with. They are:

1) Carnot
2) Reversed Carnot (Refrigeration)
3) Otto (Gasoline cycle)
4) Rankine (Power cycle)
5) Reversed Rankine (Refrigeration)

Morning (a.m.) questions will generally be single process questions. Afternoon questions could involve the analysis of a complete cycle.

Let's go over a few definitions.

For a power cycle:

$$\eta = \frac{\text{Net Work}}{\text{Heat Supplied}} = \frac{W_{net}}{Q_H}$$

For the Carnot cycle

$$\eta_c = \frac{Q_H - Q_L}{Q_H} = \frac{T_H - T_L}{T_H} = 1 - \frac{T_L}{T_H}$$

(TEMPS. IN ABSOLUTE VALUES!)

For a reversed Carnot Cycle (Refrigeration)

Coef. of Performance, COP = $\dfrac{\text{Desired Effect}}{\text{Energy that Costs}}$

For Cooling

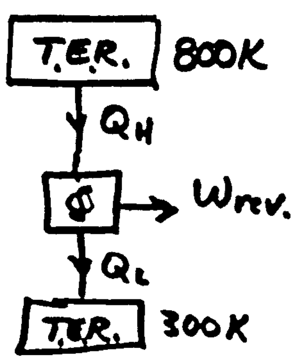
$$(COP)_C = \frac{Q_L}{(Q_H - Q_L)} = \frac{T_L}{(T_H - T_L)}$$

For Heating

$$(COP)_H = \frac{Q_H}{(Q_H - Q_L)} = \frac{T_H}{(T_H - T_L)}$$

EXAMPLE #9 Calculate the thermal efficiency for a Carnot engine operating between the temperatures of 800K and 300K.

SOLUTION:



NOTE: A T.E.R. is a Thermal Energy Reservoir

$$\eta_c = f(T_H, T_L)$$

$$\eta_c = \frac{T_H - T_L}{T_H} = \frac{800 - 300}{800}$$

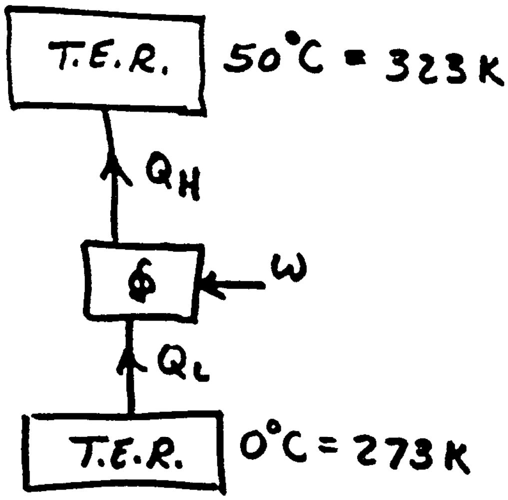
$$\boxed{\eta_c = 0.625}$$

EXAMPLE #10 A reversed Carnot cycle operates between 50°C and 0°C. Calculate the following:

a) $COP_{cooling}$, COP_c

b) $COP_{heating}$, COP_{hp}

SOLUTION: (Don't forget to convert to absolute temperatures!)



a) $$COP_c = \frac{Q_L}{(Q_H - Q_L)} = \frac{T_L}{(T_H - T_L)}$$

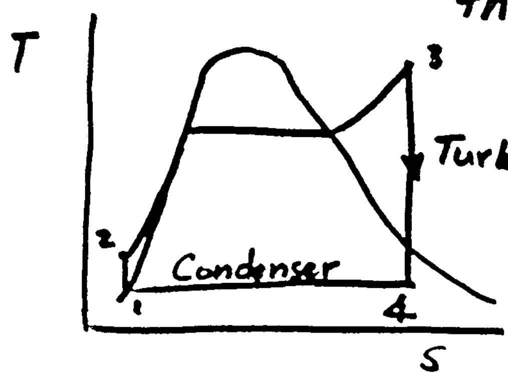
$$COP_c = \frac{273}{(323-273)} = \boxed{5.5}$$

b) $$COP_{hp} = \frac{Q_H}{(Q_H - Q_L)} = \frac{T_H}{(T_H - T_L)}$$

$$COP_{hp} = \frac{323}{(323-273)} = \boxed{6.5}$$

Rankine Cycle (Simple Power Plant)

EXAMPLE # 11 A simple power plant operates on a cycle as shown on the T-s diagram, with the pressures and temperatures as indicated around the cycle.



1-2 pump
2-3 boiler
3-4 turbine
4-1 condenser

State 1 - $P_1 = 70.14$ kPa
Saturated liquid

State 2 - $P_2 = 10$ MPa

State 3 $P_3 = 10$ MPa
$T_3 = 520\ °C$
$s_3 = 6.6622 \frac{kJ}{kg\ K}$
$h_3 = 3425.1 \frac{kJ}{kg}$

State 4 $P_4 = 70.14$ kPa
$s_4 = s_3 = 6.6622 \frac{kJ}{kg\ K}$

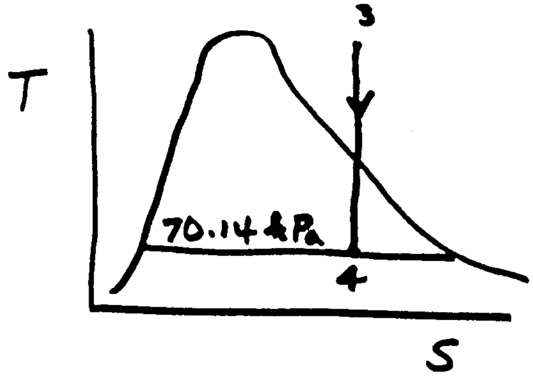
Note: When properties such as those in State 3 are not in the supplied reference, they will be given in the stem.

Determine the following:

a) Turbine Work
b) Pump Work
c) Heat Supplied in Boiler
d) Cycle Efficiency
e) MW output of turbine for $\dot{m} = 100$ kg/s
f) Isentropic η of turbine if pt. 4 is 70.14 kPa, sat. vapor

SOLUTION:

a) Turbine Work, $w_t = (h_3 - h_4)$, $\frac{kJ}{kg}$



Since pt. 4 lies in the mixture region, we have to find the quality, x, to find h_4.

We find x from the entropy, because it is given that $s_3 = s_4 = 6.6622$ kJ/(kg·K)

p. 48

@ 70.14 kPa

$s_f = 1.1925$	$h_f = 376.92$
$s_{fg} = 6.2866$	$h_{fg} = 2283.2$
$s_g = 7.4791$	$h_g = 2660.1$

0.43

$$s = x\, s_{fg} + s_f$$

$$x = \frac{s - s_f}{s_{fg}} = \frac{6.6622 - 1.1925}{6.2866} = \underline{0.87}$$

$$h_4 = x\, h_{fg} + h_f \;;\; h_4 = 0.87(2283.2) + 376.92$$

$$\underline{h_4 = 2363.4\ \text{kJ/kg}}$$

a) $$w_t = (h_3 - h_4) = (3425.1 - 2363.4) = \boxed{1061.8\ \frac{kJ}{kg}}$$

b) Pump Work, kJ/kg

Since water is incompressible, i.e., const. volume,

$$w_p = -v\,(p_2 - p_1) \qquad \text{p. 44}$$

$$v = 0.001036\ \text{m}^3/\text{kg} \qquad \text{p. 48}$$

$$w_p = -(.001036)(10\times10^3\ \text{kPa} - 70.14\ \text{kPa}) = \boxed{-10.3\ \frac{kJ}{kg}}$$

(negative work)

c) Heat Supplied by Boiler, $\frac{kJ}{kg}$

$$Q_B = (h_3 - h_2) = (3425.1 - h_2)$$

Although the pump work is small, you find

$$h_2 \cong h_1 + w_p = 376.92 + 10.3 = 387.2 \ \frac{kJ}{kg}$$

$$Q_B = (3425.1 - 387.2) = \boxed{3037.9 \ \frac{kJ}{kg}}$$

(Q_B is positive because it is heat added to cycle.)

d) Cycle efficiency, η

$$\eta = \frac{\text{Net Work}}{Q_B} = \frac{w_t - w_p}{Q_B}$$

$$\eta = \frac{(1061.8 - 10.3)}{3037.9} = \boxed{0.346 \approx 0.35}$$

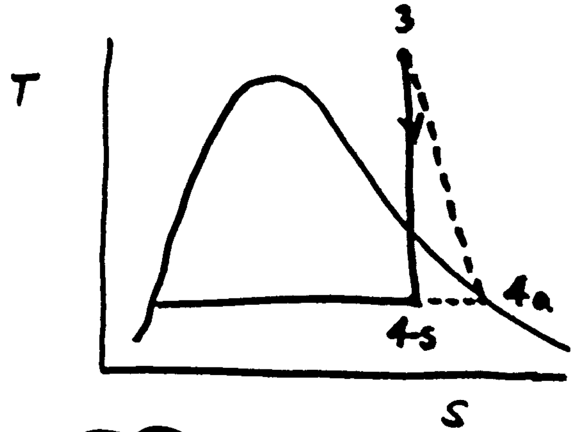
e) Power Output if $\dot{m} = 100$ kg/s

$$kw = \dot{m}\, w_t = (100 \ \text{kg/s})(1061.8) \frac{kJ}{kg}$$

$$kw = 106{,}180$$

$$MW = \frac{kw}{1000} = \boxed{106 \ MW}$$

f) Isentropic efficiency if point 4 is expanded to 70.14 kPa and saturated vapor



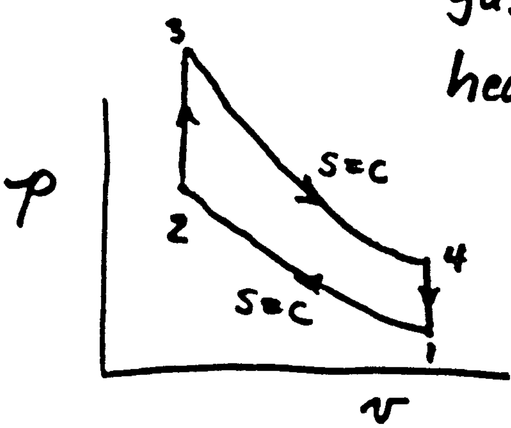
p. 48, $h_{4a} = 2660.1 \frac{kJ}{kg}$
@ 70.14 kPa

p. 44 $\quad \eta_t = \frac{(h_i - h_e)}{(h_i - h_{es})} = \frac{(h_3 - h_{4a})}{(h_3 - h_{4s})} = \frac{(3425.1 - 2660.1)}{(3425.1 - 2363.4)}$

$$\boxed{\eta_t = 0.72}$$

Otto Cycle

EXAMPLE #12 Air is the working fluid for an Otto cycle. Assume ideal gas relationships, constant specific heats, and $v_1/v_2 = 10$.



Otto Cycle = 2 isentropic processes and 2 Const. Volume processes

a) Determine the cycle efficiency

b) Find T_2 in °C if $T_1 = 27$°C

c) If $T_3 = 827$°C, find the heat added in kJ/kg.

SOLUTION: a) From p. 47, Otto Cycle

$$\eta = 1 - r^{1-k} \qquad \text{where } r = \frac{v_1}{v_2} = 10$$

$k = 1.4$ p. 52

$$\eta = 1 - (10)^{1-1.4} = 1 - (10)^{-0.4} = \boxed{0.60}$$

b) Find T_2 for the isentropic compression from State 1 to State 2.

Use the isentropic expressions on p. 43

$$T_1 v_1^{k-1} = T_2 v_2^{k-1}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{v_1}{v_2}\right)^{k-1} = (10)^{1.4-1} = (10)^{0.4}$$

$$\left(\frac{T_2}{T_1}\right) = 2.51 \;;\; T_2 = (273+27)(2.51)$$

$$\boxed{T_2 = 753.6\ K} \qquad \boxed{T_2 = 481\,^\circ C}$$

c) Find q_{added} in kJ/kg if $T_3 = 827\,^\circ C$

1^{st} Law $q - \cancel{w}^{\,0} = \Delta u$ $\qquad w = 0$ from 2-3, since $v = C$

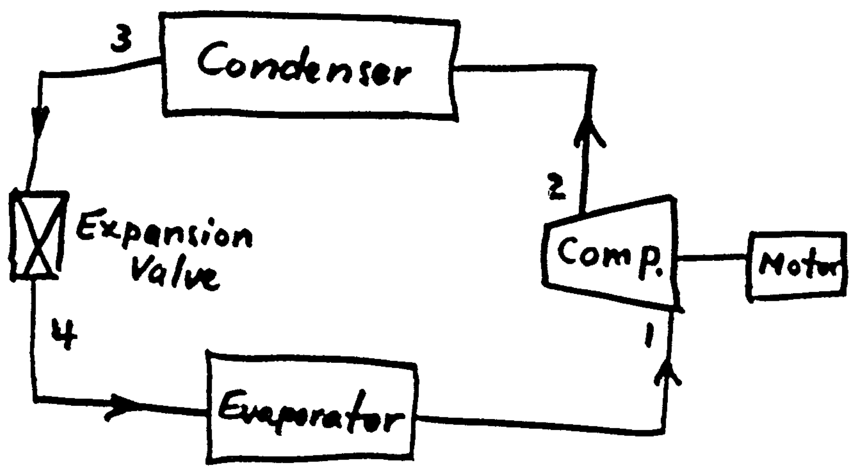
$c_v = 0.718$ p. 52

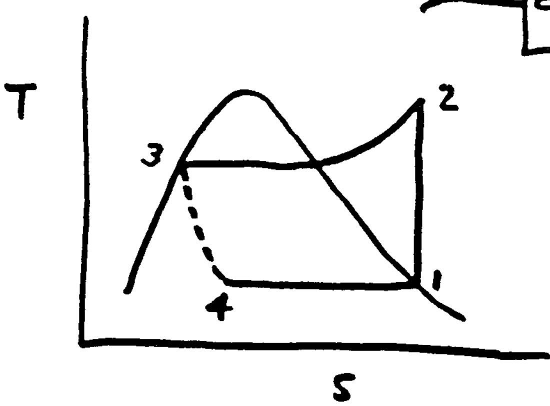
$$q = \Delta u = c_v(\Delta T)$$

$$q = 0.718\,(827 - 481) = \boxed{248\ \frac{kJ}{kg}}$$

Refrigeration Cycle (Reverse Rankine)

Real refrigeration cycles will involve R-134a and will consist of the following cycle.



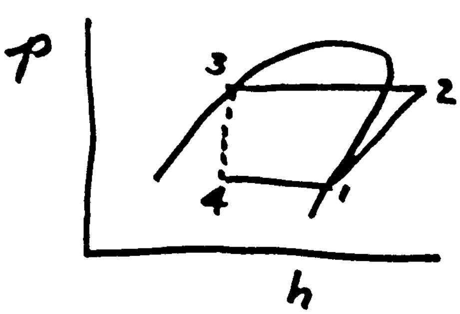


1-2 Isentropic Compression

2-3 Const. pressure cooling in condenser

3-4 Expansion through throttling device - const. enthalpy

4-1 Const. pressure evaporation in evap.



p. 50 has the p-h diagram for R-134a. A p-h diagram is helpful in the cycle analysis.

EXAMPLE #13: A refrigerant cycle operates between 0.2 MPa and 1.0 MPa. Assume the R-134a leaves the condenser as saturated liquid and enters the compressor as saturated vapor.

Calculate:
a) Evaporator cooling, $\frac{kJ}{kg}$
b) Heat rejected in condenser, $\frac{kJ}{kg}$
c) Compressor work, $\frac{kJ}{kg}$
d) COP for cooling
e) COP for heating

SOLUTION: Find enthalpies around the cycle. (Read the p-h diagram as accurately as you can!)

$h_1 = 394 \frac{kJ}{kg}$ (p.50, sat. vapor, 0.2 MPa)

$h_2 = 427 \frac{kJ}{kg}$ (p.50, const. entropy, p = 1 MPa)

$h_3 = h_4 = 258 \frac{kJ}{kg}$ (p.50, const. pressure = 1 MPa, sat. liquid)

a) Cooling in Evaporator $= (h_1 - h_4) = (394 - 258)$

$$\boxed{q_{evap} = 136 \frac{kJ}{kg}}$$

b) Heat Rejected in Condenser = $(h_2 - h_3)$

$$q_{cond} = (427 - 258) = \boxed{169 \ \frac{kJ}{kg}}$$

c) Compressor Work = $(h_2 - h_1) = (427 - 394)$

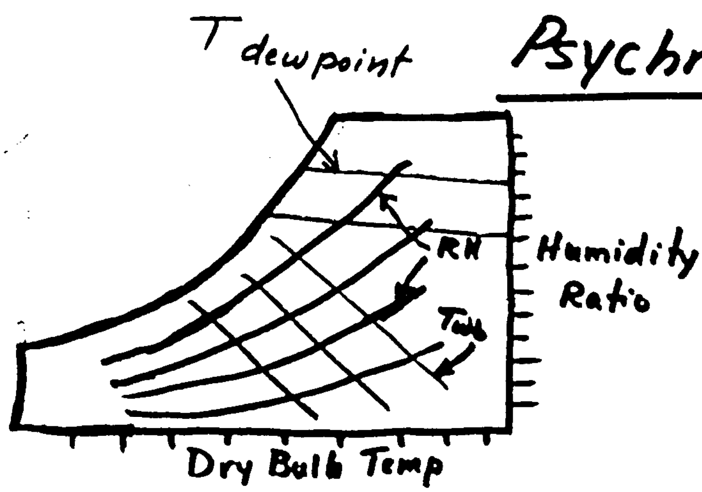
$$\boxed{W_{comp.} = 33 \ \frac{kJ}{kg}}$$

d) $$COP_{Cooling} = \frac{Q_{evap}}{W_{comp}} = \frac{136}{33} = \boxed{4.1}$$

e) $$COP_{Heating} = \frac{Q_{cond}}{W_{comp}} = \frac{169}{33} = \boxed{5.1}$$

Note that COP for heating is higher than COP for cooling. The difference is the compressor work.

Psychrometric Relations



Psychrometric charts can be used for air-water mixtures.

EXAMPLE # 14 Air conditioned rooms are typically maintained at 23°C, 50% relative humidity.

Determine:
a) Wet bulb temp. of space
b) Dew point temp. of space
c) Grams of moisture per kg of dry air

SOLUTION: These are direct look-ups on the psychrometric chart.

a) T_{wb} is 16.3°C. (Follow the T_{db} temp. of 23°C up to the 50% RH line.) The intersection gives the T_{wb}.

b) $T_{dew\,pt.} = 11.8°C$ ($T_{dew\,pt}$ can be found from a horizontal line to the saturation temperature.)

c) Humidity ratio $\cong$ 9 grams moisture per kg of dry air.

Partial Pressures and Psychrometric Relations

<u>EXAMPLE #15</u> $100\,m^3$ of an air-water mixture are at a pressure of 100 kPa, 35°C, and 70% relative humidity.

Find the a) humidity ratio, ω
b) dew point
c) mass of water vapor
d) mass of dry air

1. Use partial pressure equations and steam tables.

2. Use Psychrometric Chart and relations

<u>SOLUTION</u>: p. 45 Relative humidity, ϕ

$$\phi = \frac{m_v}{m_g} = \frac{P_v}{P_g} = 0.70$$

where m_v = mass vapor
m_g = mass vapor at saturation
P_v = pressure of vapor
P_g = pressure of vapor at sat. temp.

From steam tables, p. 48, at 35°C

$$P_g = 5.628\ kPa$$

$$P_v = 0.70\,(5.628) = 3.94\text{ kPa}$$

(Partial pressure of water vapor.)

$$P_{air} = 100\text{ kPa} - 3.94 = 96.06\text{ kPa}$$

a) humidity ratio, $\omega = 0.622\,\dfrac{P_v}{P_a} = 0.622\left(\dfrac{3.94}{96.06}\right)$

$$\boxed{\omega = 0.0255 = \frac{m_v}{m_a}}$$

b) Dew Point is found from the saturation temperature at a pressure of 3.94 kPa. (from steam tables, p. 48)

You have to interpolate between

T		P	
25°C		3.169 kPa	
	28.6°C		3.94 kPa
30°C		4.246 kPa	

$$\boxed{T_{dew\ point} = 28.6\,°C}$$

c) Mass of water vapor $\longrightarrow$ ideal gas law

$$P_v V = m_v R T$$

$$m_v = \frac{P_v V}{RT} = \frac{(3.94)(100)}{\left(\frac{8.314}{18}\right)(308)} = 2.77\text{ kg}$$

d) Mass of air (from ω & m_v OR $P_a V = m_a R T$)

$$\omega = \frac{m_v}{m_a}\;;\quad m_a = \frac{2.77}{0.0255} = \boxed{108.6\text{ kg}}$$

2. Now solve the same problem using the psychrometric chart and psychrometric relations.

From p.51, at 35°C and 70% RH

a) A horizontal line to the right ordinate gives $\omega = \dfrac{25.4 \text{ grams}}{\text{kg dry air}}$

or $\boxed{\omega = 0.0254}$

b) A horizontal line to the saturation curve yields $\boxed{T_{dew\ point} \cong 28.6°C}$

c) To find m_v, we have to find P_v

p.45

$$\omega = 0.622 \frac{P_v}{P_a} = 0.622 \frac{P_v}{(P - P_v)}$$

$$\omega = 0.0254 = 0.622 \frac{P_v}{(100 - P_v)}$$

$$P_v + 0.0408 P_v = 4.084$$

$$P_v = 3.92 \text{ kPa}$$

From $P_v V = m_v RT$; $\boxed{m_v = 2.7 \text{ kg}}$

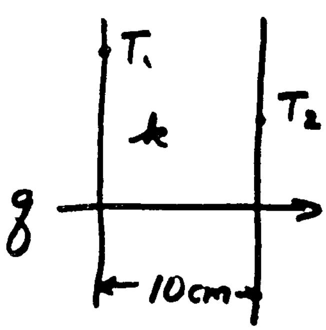
d) From $\omega = \dfrac{m_v}{m_a}$; $\boxed{m_a = 108 \text{ kg}}$

HEAT TRANSFER

Heat transfer is a part of thermodynamics on the General FE exam. The problems are limited to the concepts on p. 53 and the top of page 54.

Typical problems might include the calculation of heat transfer through a composite wall or a cylinder; convective heat transfer, or radiation using the 4^{th} power radiation law.

EXAMPLE #16 Calculate the heat transfer through a plane wall having a thermal conductivity, k, of 1 W/m·K. The face temperatures are 25°C and 0°C, respectively.

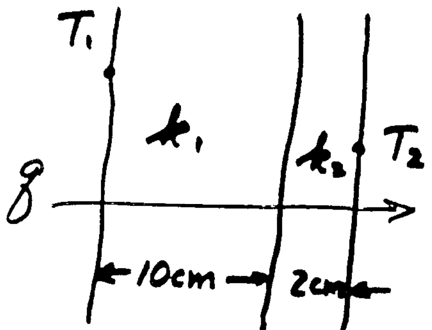


$$q = \frac{-k\,(T_2 - T_1)}{L} = \frac{-1\,(\text{W/m·K})\,(0-25)}{.1\text{ m}} = \boxed{250\ \frac{\text{W}}{\text{m}^2}}$$

If the total $Q = q(A)$ were requested, the wall Area in m^2 would be multiplied by q.

EXAMPLE #17:

For the plane wall in EXAMPLE #16, add a 2 cm insulating board with a thermal conductivity of 0.05 W/m·K. Assume the same surface temperatures.



25°C —— $\frac{L_1}{k_1}$ —— $\frac{L_2}{k_2}$ —— 0°C ($q \rightarrow$)

$\left(\frac{0.1}{1}\right)$ $\left(\frac{0.02}{0.05}\right)$

(.1) (0.4)

$$q = \frac{\Delta T}{\Sigma R}$$

$$\boxed{q = \frac{25}{0.5} = 50 \text{ W/m}^2}$$

EXAMPLE #18:

A cylindrical heat source is 1 cm diameter × 10 cm long. The temperature is maintained at 700 K. Calculate the amount of black body radiation exchanged with surroundings at 300 K.

$$q = \underbrace{\varepsilon}_{1 \text{ for a black body}} A_s \sigma \left(T_1^4 - T_0^4\right)$$

$$q = \pi(.01)(0.1)\,\cancel{m^2}\left[5.67\times10^{-8}\frac{W}{\cancel{m^2}\cancel{K^4}}\right]\left[700^4 - 300^4\right]\cancel{K^4}$$

$$\boxed{q = 41.3 \text{ watts}}$$