Fundamentals of Engineering Examination Review

Lee L. Lowery, Jr., Phd, PE

Strength of Materials

Overview

- ***In strength of materials, we are looking for basic stresses induced in structural members due to loading, and the resulting deflections.***
- ***Using derived equations, a close approximation of the stress level and deflections in the member can be found and compared with allowed values.***

Outline

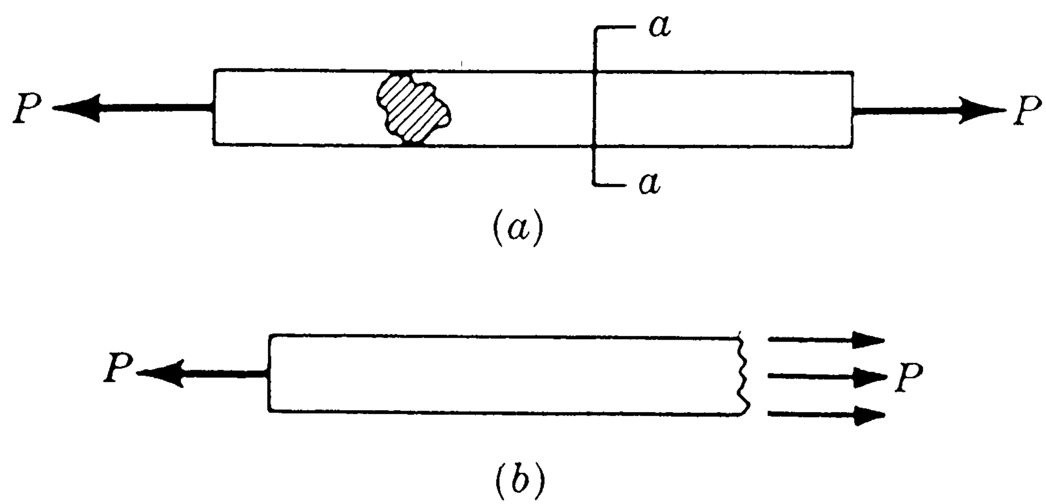
- ***Direct Stresses***
  - ***Normal***
  - ***Shear***
- ***Torsional Stresses***
- ***Bending Stresses***
- ***Pressure Vessels***
- ***Combined Stresses***
- ***Columns***

Outline (continued)

- ***Deflections***
  - ***Axial deformations***
  - ***Torsional deformations***
  - ***Beam deflections***

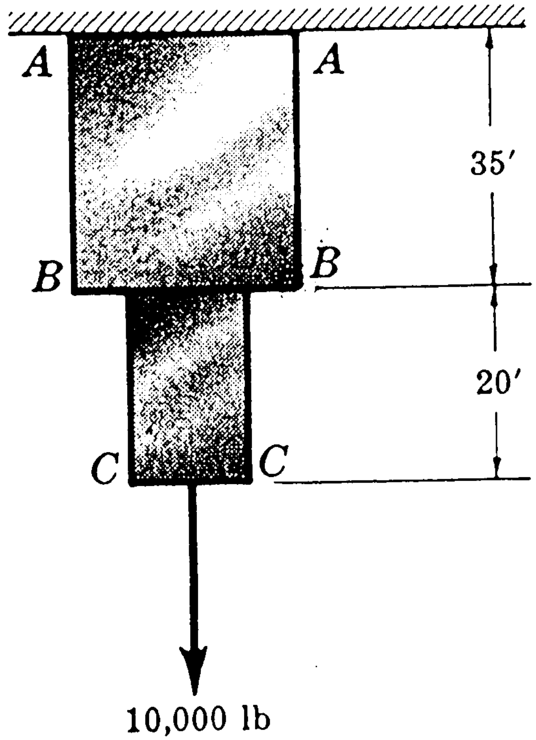
Direct Stresses

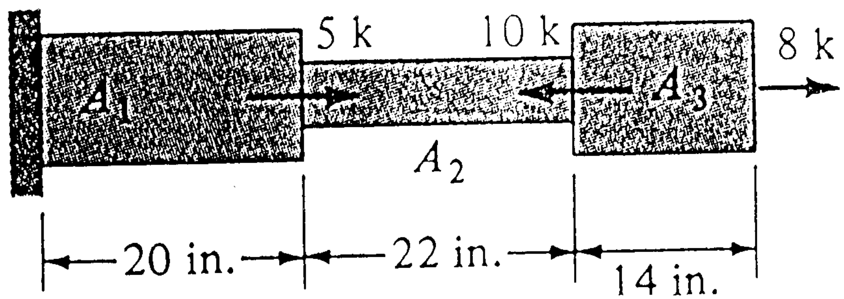
- ***Tension and Compression stresses***
  - ***where σ = P/A - stretching and squashing parallel to load and long axis of member.***
- ***Shear stresses***
  - ***where τ = P/A - shear stresses - wiping stresses - cross-shear on bolts, stresses in web of beam - stresses across the long axis of the member.***



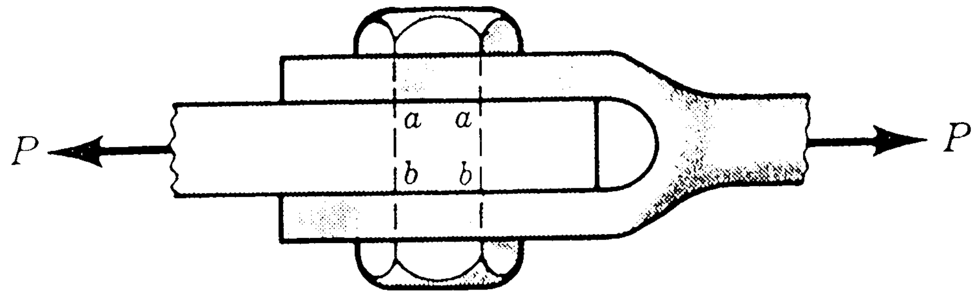
Bar in tension

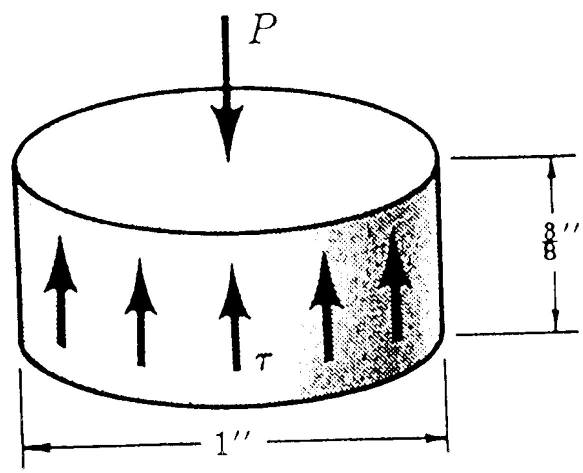
Bar in compression


A
A
35′
B
B
20′
C
C
10,000 lb



5 k
10 k
8 k
A_1
A_2
A_3
20 in.
22 in.
14 in.


P
a a
b b
P


P
$\frac{3}{8}''$
τ
1''

Longitudinal Deflections of Members

- δ_{load} = ***PL/AE where***

  P is the axial load

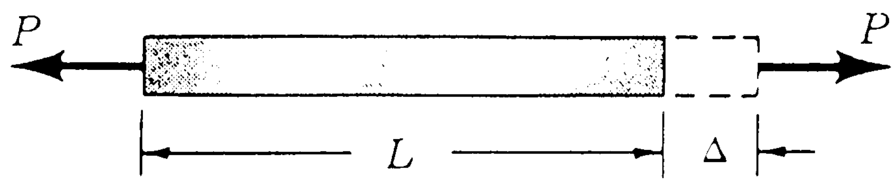
  L is the length of the member

  A is the cross-sectional area

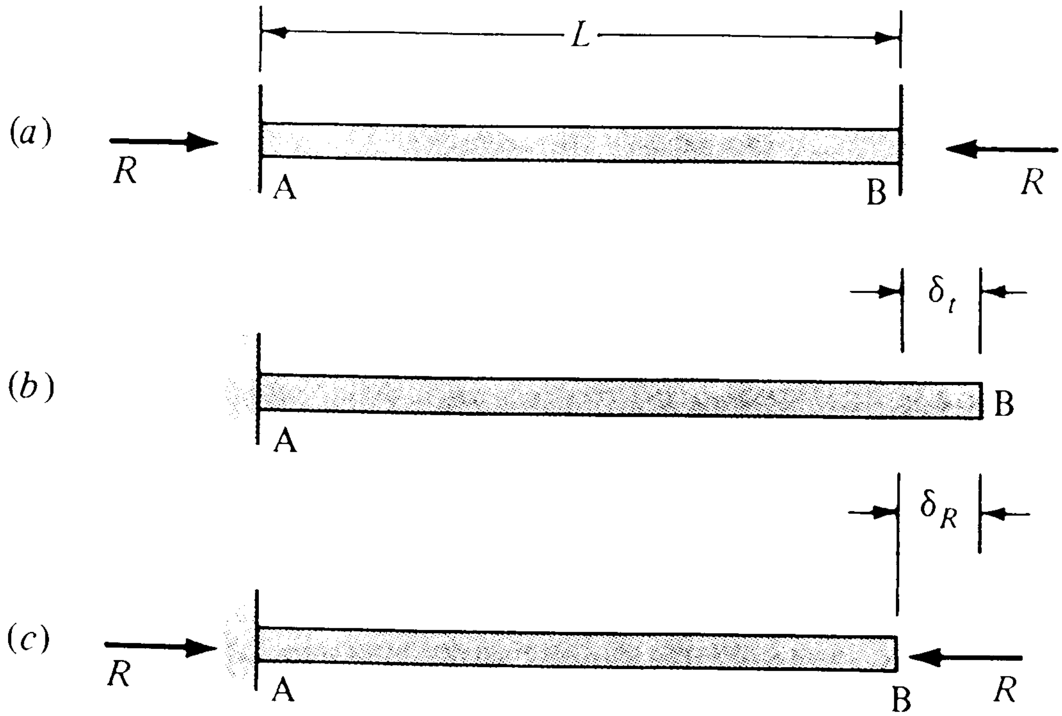
  E is the modulus of elasticity
- $\delta_{temp} = \alpha \Delta T L$ ***where***

  α ***is the coefficient of thermal expansion***

  and ΔT ***is the change in temperature***



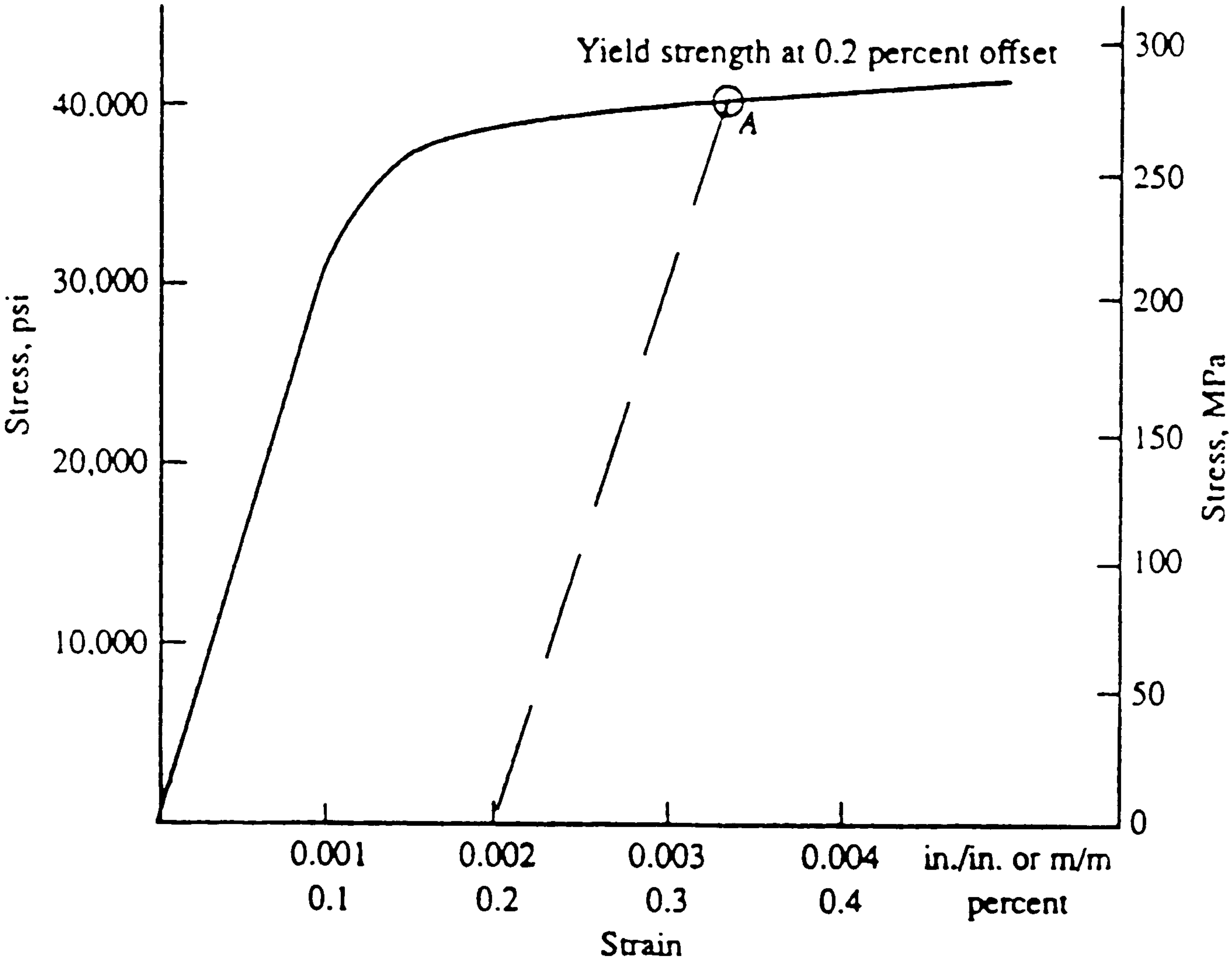
$$E = \frac{\sigma}{\epsilon} = \frac{P/A}{\Delta/L} = \frac{PL}{A\Delta} \qquad \text{or} \qquad \Delta = \frac{PL}{AE}$$


L
(a)
R
A
B
R
δt
(b)
A
B
δR
(c)
R
A
B
R

UNIAXIAL STRESS-STRAIN

Stress-Strain Curve For Mild Steel

♦



The slope of the linear portion of the curve equals the modulus of elasticity.

HOOKE'S LAW $\sigma = E\epsilon$, where

σ = stress (force per unit area),

E = modulus of elasticity (force per unit area), and

ϵ = strain (change in length per original length).

ENGINEERING STRAIN $\epsilon = \Delta L / L_o$, where 14

ϵ = engineering strain (units per unit),
ΔL = change in length (units) of member,
L_o = original length (units) of member,
ϵ_{pl} = plastic deformation (permanent), and
ϵ_{el} = elastic deformation (recoverable).

Equilibrium requirements: $\Sigma \boldsymbol{F} = 0$; $\Sigma \boldsymbol{M} = 0$

Determine geometric compatibility with the restraints.

Use a linear force-deformation relationship;

$$F = k\delta.$$

Uniaxial Loading and Deformation

$$\sigma = P/A, \quad \text{where}$$

σ = stress on the cross section,
P = loading, and
A = cross-sectional area.

$$\epsilon = \delta/L, \quad \text{where}$$

δ = axial deformation and
L = length of member.

$$E = \sigma/\epsilon = \frac{P/A}{\delta/L}$$

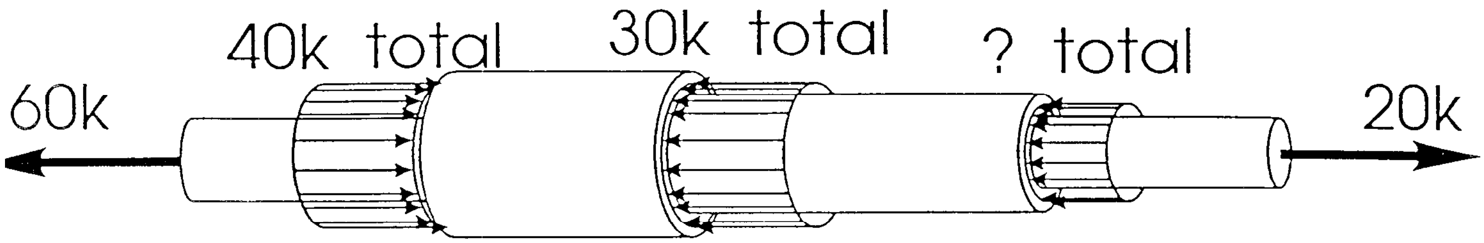
$$\delta = \frac{PL}{AE}$$

THERMAL DEFORMATIONS

$$\delta_t = \alpha L (t - t_o), \text{ where}$$

δ_t = deformation caused by a change in temperature,
α = temperature coefficient of expansion,
L = length of member,
t = final temperature, and
t_o = initial temperature.

How much does the bar shown elongate A to D?



Bar	Diameter	Ex10^3ksi	Length
A	3"	30	40"
B	4"	10	30"
C	2"	20	10"

Shear Stress-Strain

$$\gamma = \tau / G, \quad \text{where}$$

γ = shear strain,
τ = shear stress, and
G = *shear modulus* (constant in linear force-deformation relationship).

$$G = \frac{E}{2(1 + \nu)} \quad , \text{ where}$$

ν = *Poisson's ratio*,
= – (lateral strain)/(longitudinal strain).

MATERIAL PROPERTIES

Quantity		Symbol	Value	Units
modulus of elasticity, steel	metric	E_s	2.1×10^{11}	Pa
modulus of elasticity, aluminum	metric	E_a	6.9×10^{10}	Pa
modulus of elasticity, steel	USCS	E_s	30×10^{6}	psi
modulus of elasticity, aluminum	USCS	E_a	10×10^{6}	psi
shear modulus, steel	metric	G_s	8.3×10^{10}	Pa
shear modulus, aluminum	metric	G_a	2.8×10^{10}	Pa
shear modulus, steel	USCS	G_s	12×10^{6}	psi
shear modulus, aluminum	USCS	G_a	4×10^{6}	psi

Strain--General Case

$$\epsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + \sigma_z)]$$
$$\epsilon_y = (1/E)[\sigma_y - \nu(\sigma_z + \sigma_x)]$$
$$\epsilon_z = (1/E)[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

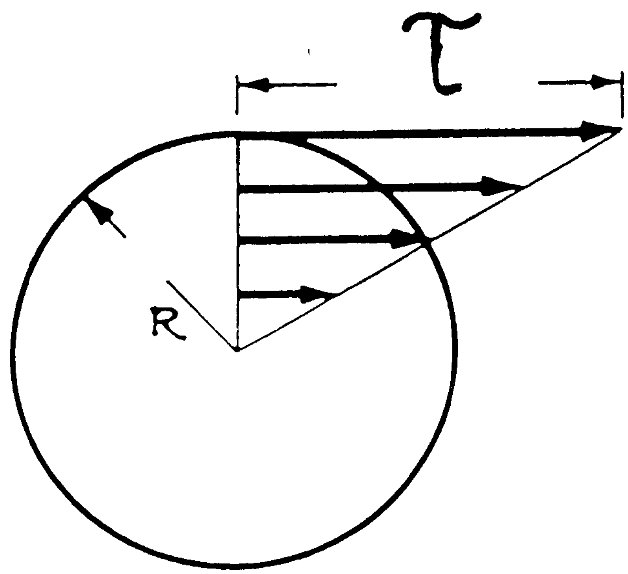
$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

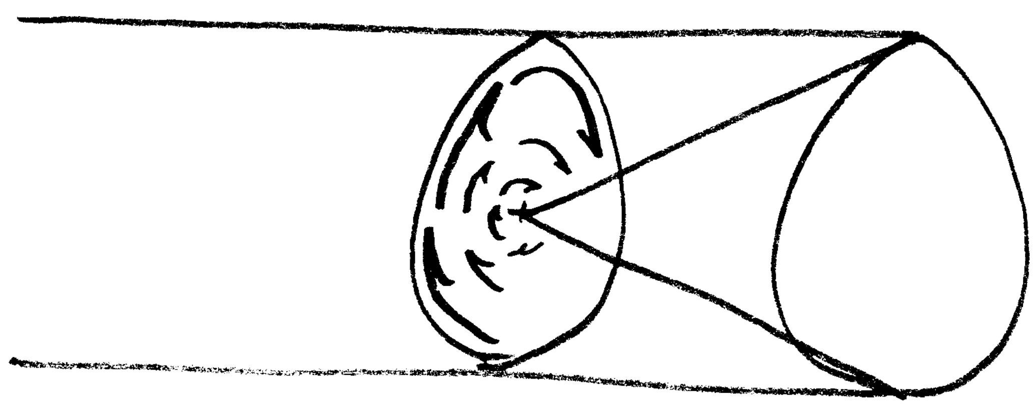
$$\gamma_{zx} = \frac{\tau_{zx}}{G}\ ,$$ where

$\epsilon_x, \epsilon_y, \epsilon_z$ = normal strain,
$\sigma_x, \sigma_y, \sigma_z$ = normal stress,
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$ = shear strain,
$\tau_{xy}, \tau_{yz}, \tau_{zx}$ = shear stress,
E = modulus of elasticity,
G = shear modulus, and
ν = Poisson's ratio.

Torsional Stresses

- $\tau = Tr / J$
  - —*where* τ *is the shear stress*
  - —*T is the applied torque*
  - —*r is the outer radius of the bar*
  - —$J = \pi (r_{outer}^{4} - r_{inner}^{4}) / 2$ (PG 26)
  - —*Shear stress is across the end face of the bar, a maximum on the outside fiber, and zero at the center of the bar. The equation applies only to circular rods and pipes.*


τ
R



$$\gamma_{\phi z} = \lim_{\Delta z \to 0} r(\Delta \phi / \Delta z) = r(d\phi/dz)$$

The shear strain varies in direct proportion to the radius, from no strain at the center to a greatest strain at the outside of a circular shaft. $d\phi/dz$ is the twist per unit length or the rate of twist.

$$\tau_{\phi z} = G\gamma_{\phi z} = Gr(d\phi/dz)$$

$$T = G(d\phi/dz)\int_A r^2 dA = GJ(d\phi/dz),$$

where

J = *polar moment of inertia* (see table at end of **DYNAMICS** section).

$$\phi = \int_0^L \frac{T}{GJ}\,dz = \frac{TL}{GJ}, \text{ where}$$

ϕ = total angle (radians) of twist,
T = moment or torque, and
L = length of shaft.

$$\tau_{\phi z} = Gr[T/(GJ)] = Tr/J$$

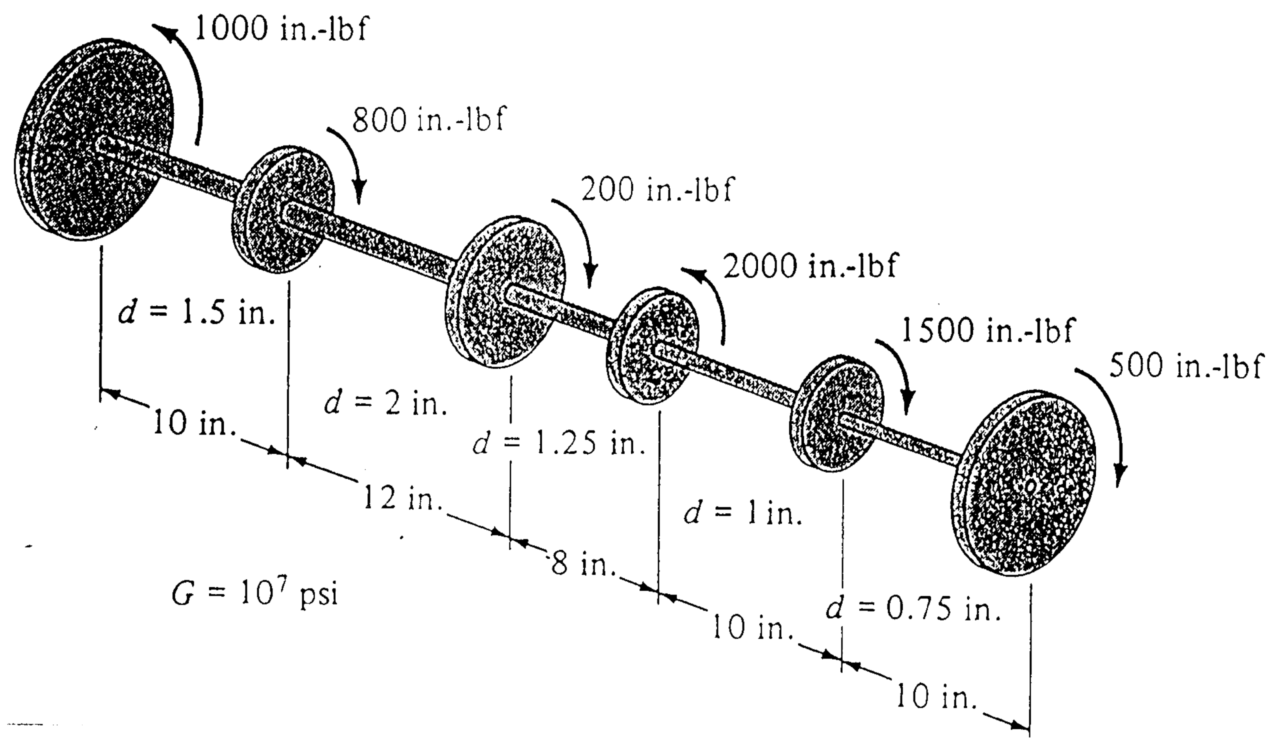
$$\frac{T}{\phi} = \frac{GJ}{L}, \text{ where}$$

T/ϕ gives the *twisting moment per radian of twist*. This is called the *torsional stiffness* and is often denoted by the symbol k or c.

For Hollow, Thin-Walled Shafts

$$\tau = \frac{T}{2A_m t}, \text{ where}$$

t = thickness of shaft wall and
A_m = the total mean area enclosed by the shaft measured to the midpoint of the wall


1000 in.-lbf
800 in.-lbf
200 in.-lbf
2000 in.-lbf
1500 in.-lbf
500 in.-lbf
d = 1.5 in.
d = 2 in.
d = 1.25 in.
d = 1 in.
d = 0.75 in.
10 in.
12 in.
8 in.
10 in.
10 in.
G = 10^7 psi

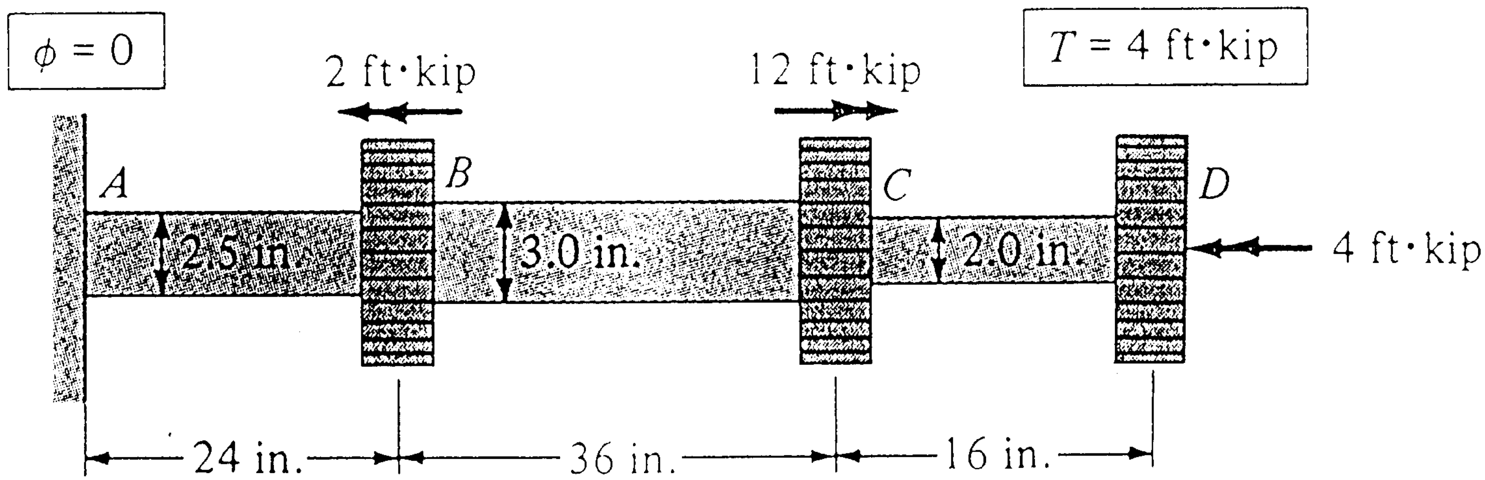


Figure 6.26 System of shafts and gears

Deflections of Torsionally Loaded Circular Shafts

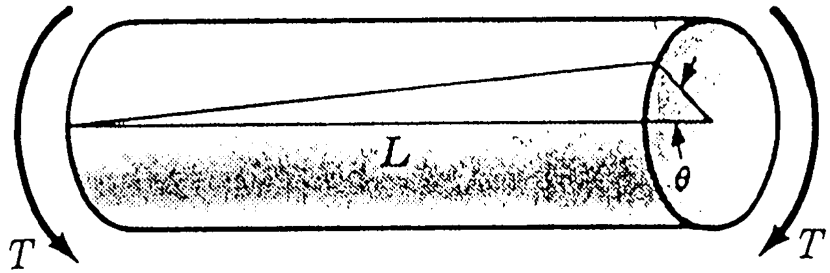
ϕ = TL/GJ = rotation between any two points on the shaft (radians), and

T = the applied torque

L = the length of the shaft

G = the shear modulus of elasticity

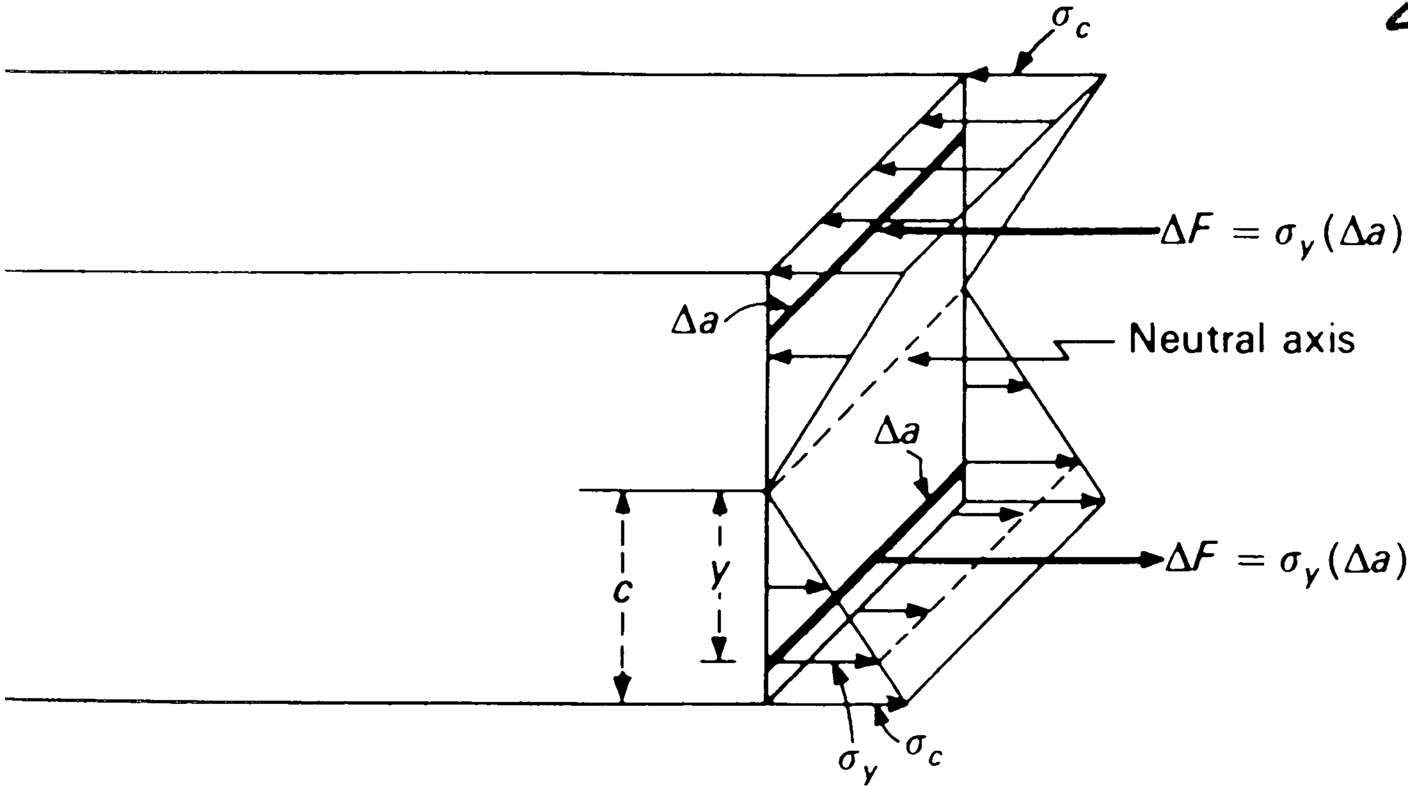
J = the polar moment of inertia of the shaft = $\pi\,(r_{outer}^4 - r_{inner}^4) / 2$



$$\theta = \frac{TL}{GJ}$$

Bending Stresses

- $\sigma = Mc / I$
  - *where σ is the bending stress in the beam*
  - *M is the applied moment*
  - *c is the distance from the neutral axis to an outside fiber on the beam*
  - *I is the moment of inertia of the beam*
  - *Stresses are normal (tensile or compressive) and are zero at the neutral axis and maximum on the outside fibers of the beam.*

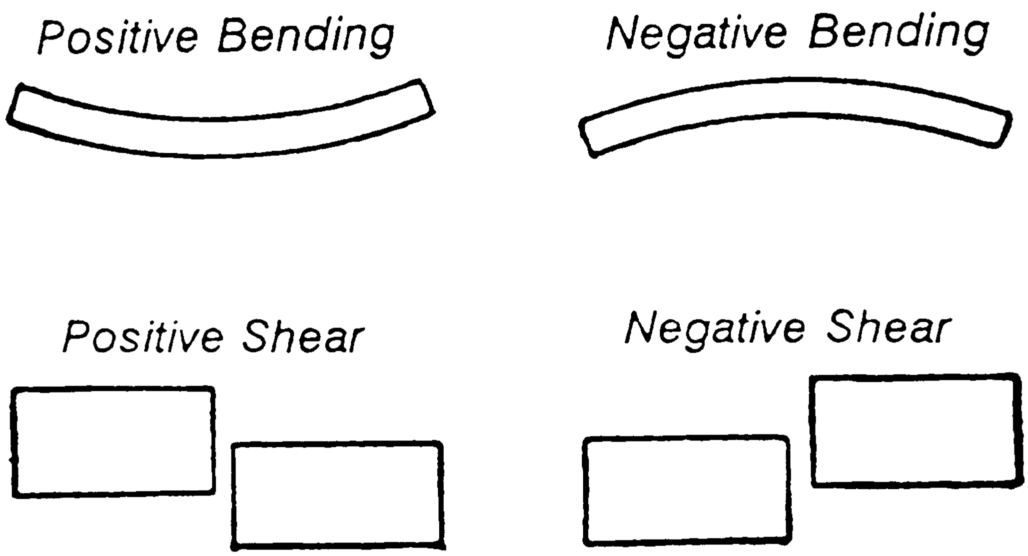


σ_c
$\Delta F = \sigma_y(\Delta a)$
Δa
Neutral axis
Δa
c
y
$\Delta F = \sigma_y(\Delta a)$
σ_y
σ_c

Shearing Force and Bending Moment Sign Conventions

1. The bending moment is *positive* if it produces bending of the beam *concave upward* (compression in top fibers and tension in bottom fibers).
2. The shearing force is *positive* if the *right portion of the beam tends to shear downward with respect to the left*.

•



The relationship between the load (w), shear (V), and moment (M) equations are:

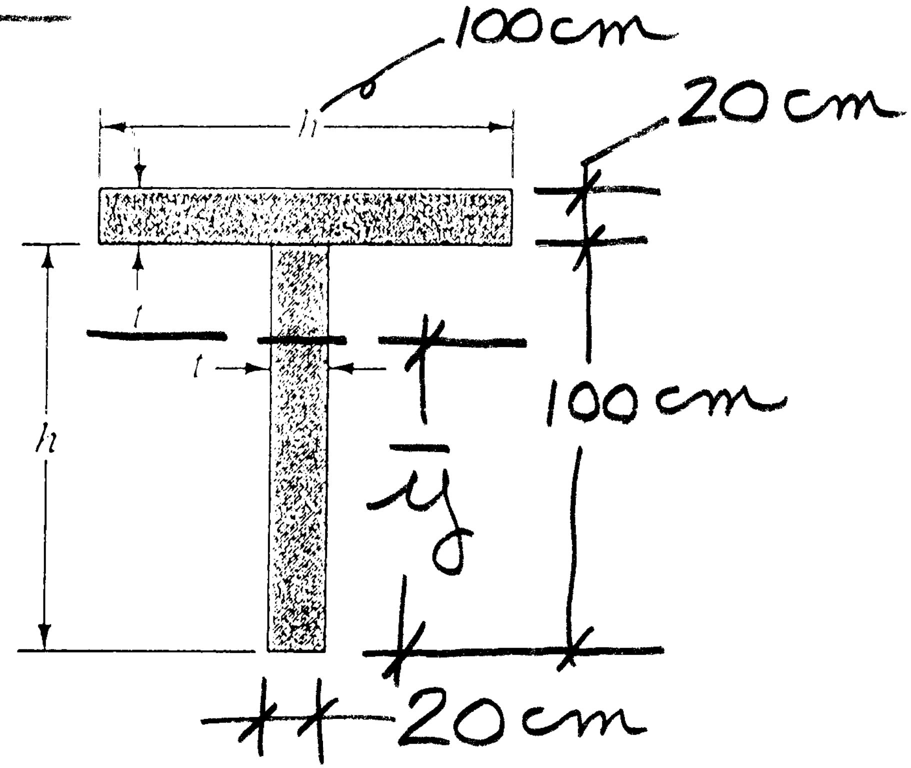
$$-w = dV(x)/dx; \quad V = dM(x)/dx$$

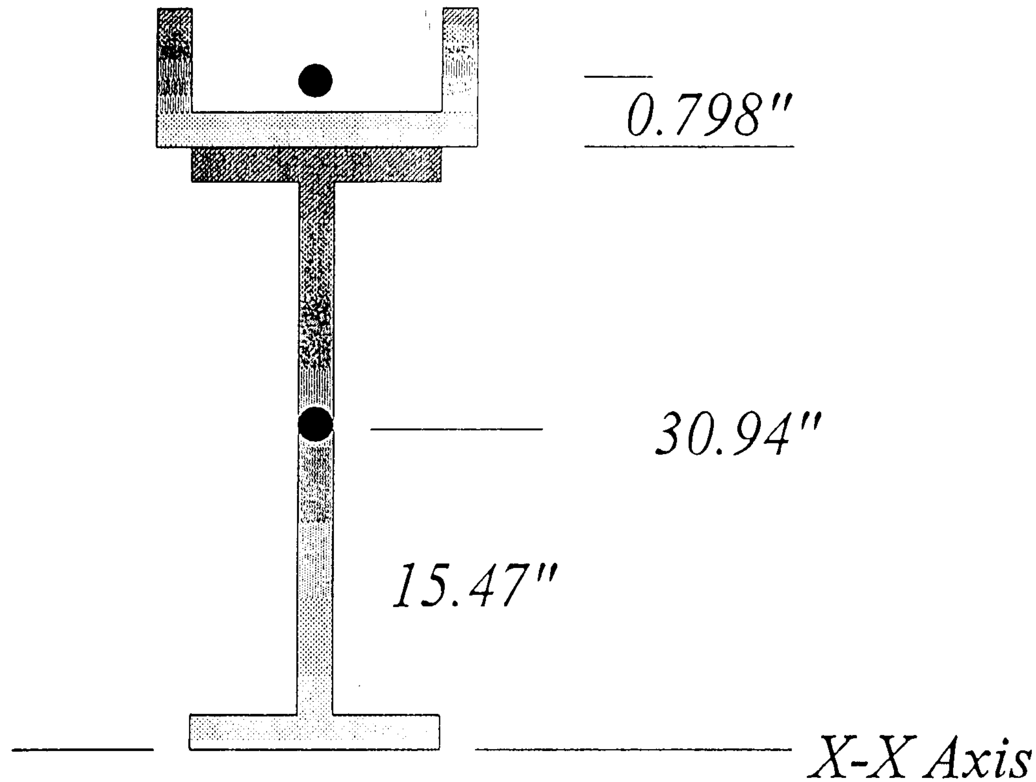
$$V_2 - V_1 = \int_{x_1}^{x_2} w(x)\,dx$$

$$M_2 - M_1 = \int_{x_1}^{x_2} V(x)\,dx$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + \cdots}{A_1 + A_2 + A_3 + \cdots}$$

$$I_{TOTAL} = \sum I_o + \sum A d^2$$



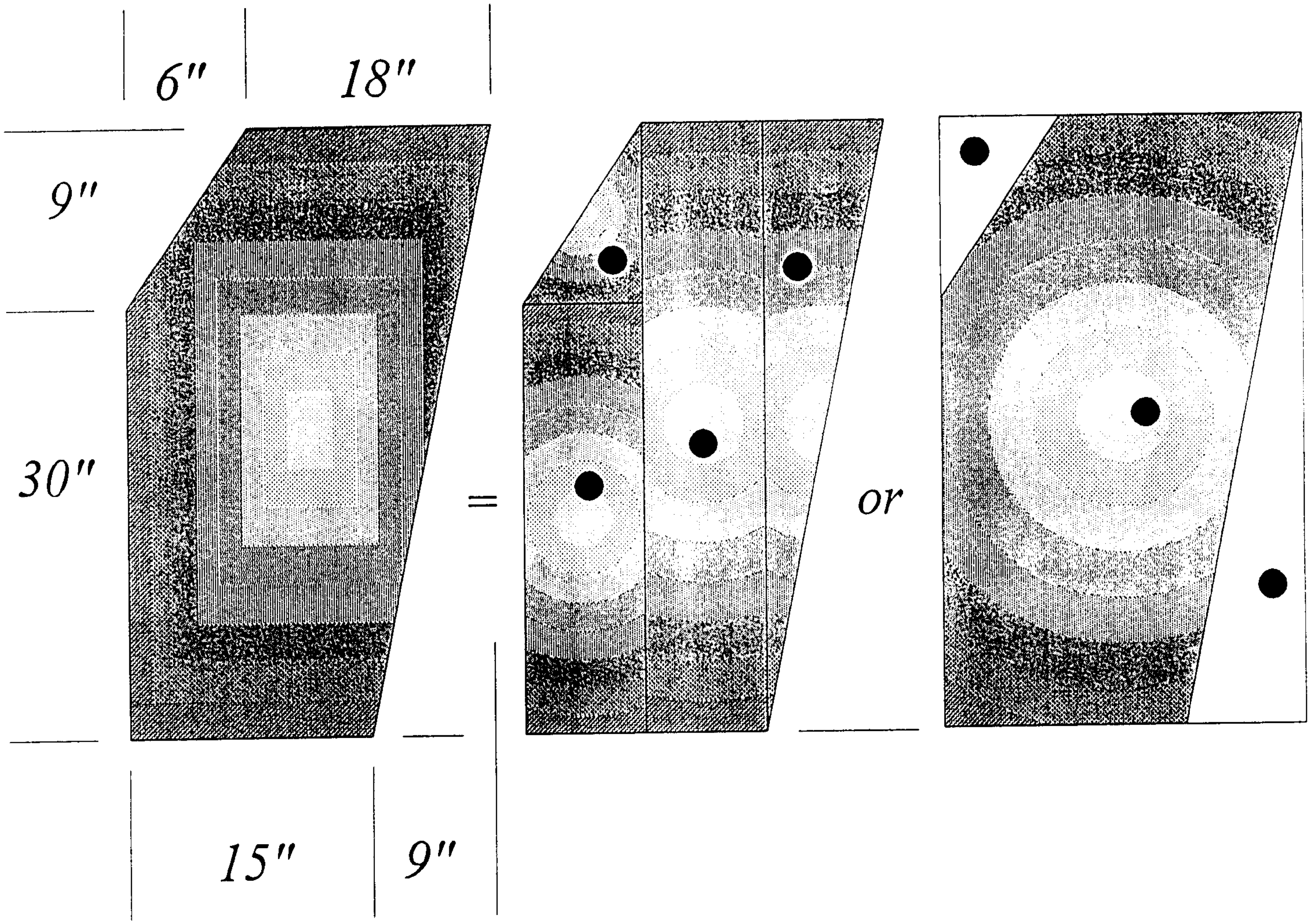


Wide flange is W30x211:

Area = 62 in^2, Ixx = 10300 in^4

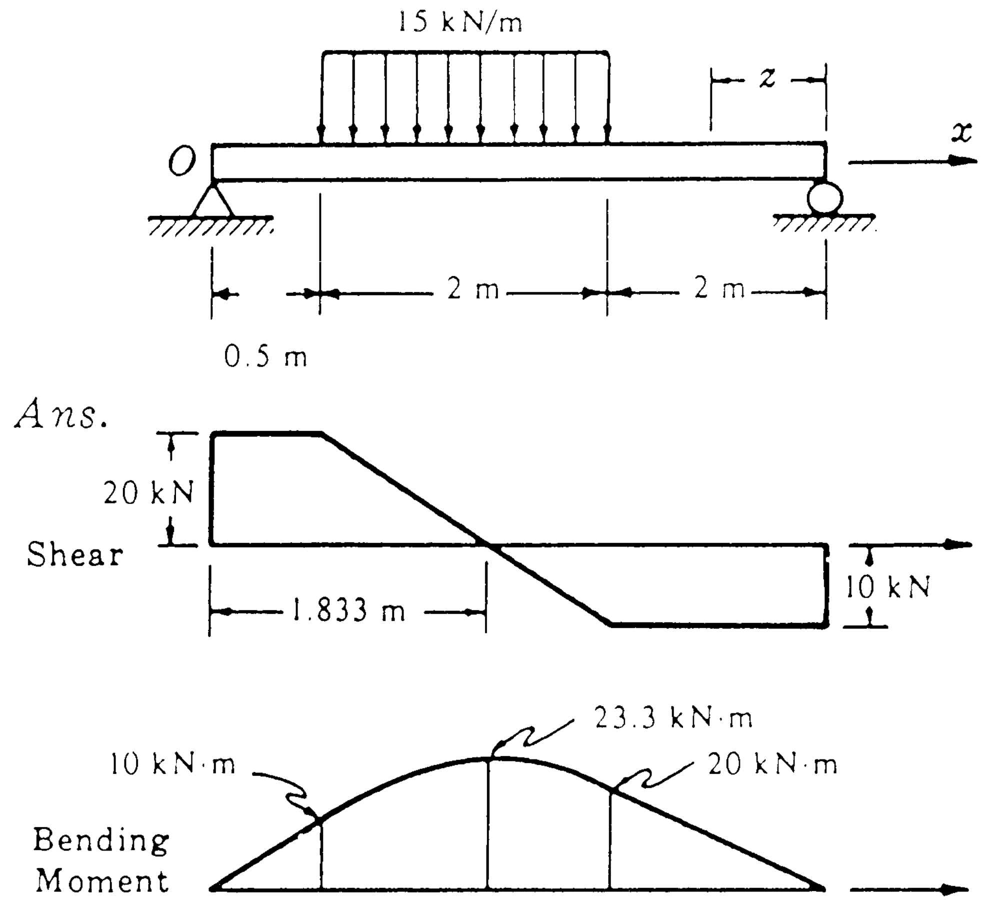
Channel is C15x50:

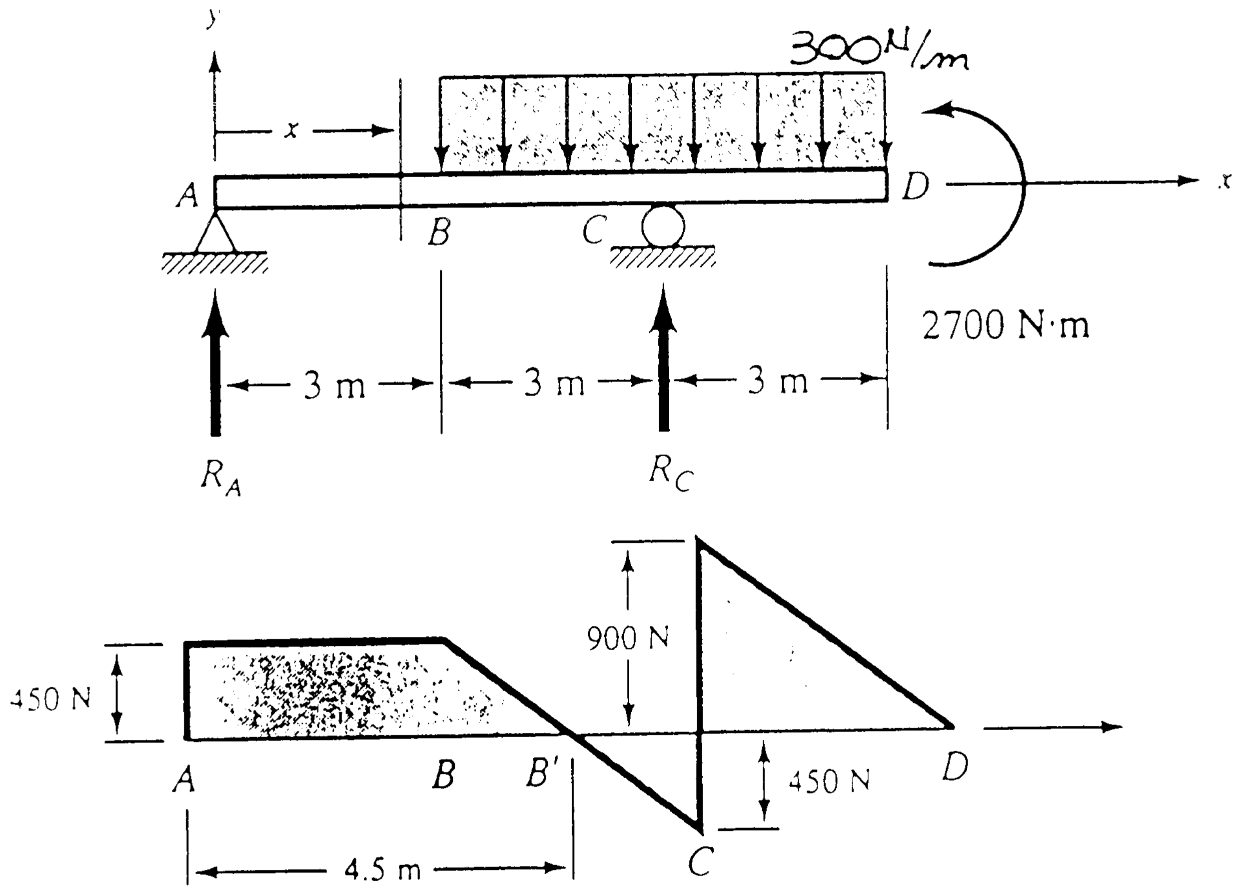
Area = 14.7 in^2, Iyy = 11 in^4


6"
18"
9"
30"
=
or
15"
9"

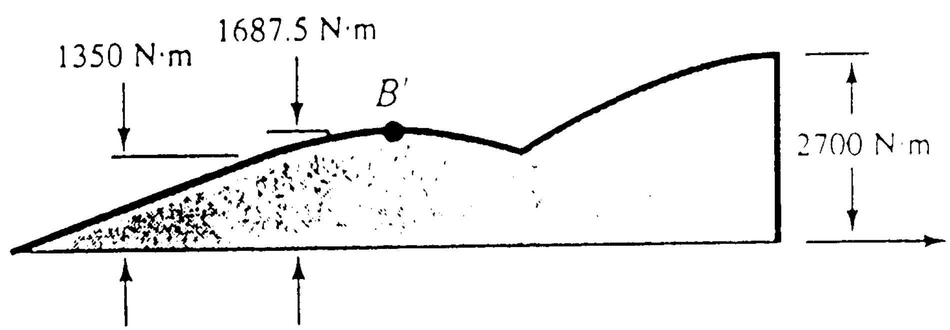
Shear and Bending Moment Diagrams

- ***Use statics to solve for reactions.***
- ***The area under any diagram gives the change in value on the next diagram.***
- ***The value on any diagram gives the slope of the next diagram.***
- ***If load is uniform constant, xbar = the starting shear (from shear diagram) / the load rate (from load diagram.)***

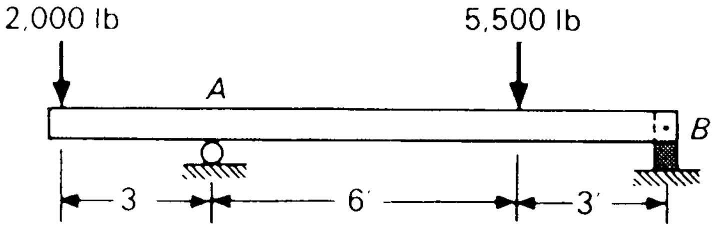

15 kN/m
z
x
O
2 m
2 m
0.5 m
Ans.
20 kN
Shear
1.833 m
10 kN
23.3 kN·m
10 kN·m
20 kN·m
Bending
Moment



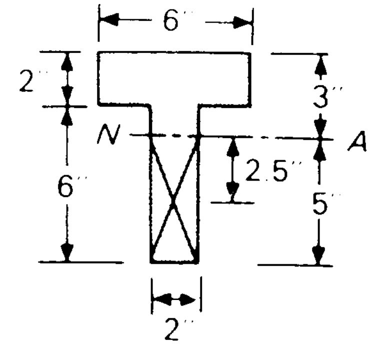
(a) Shear



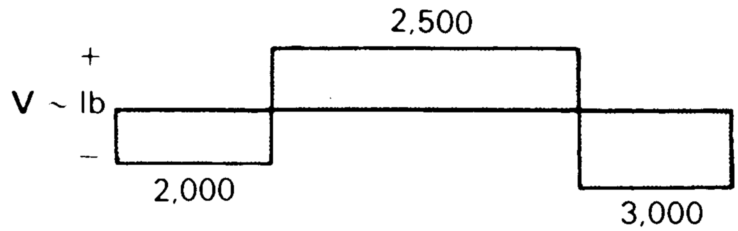
(b) Bending Moment



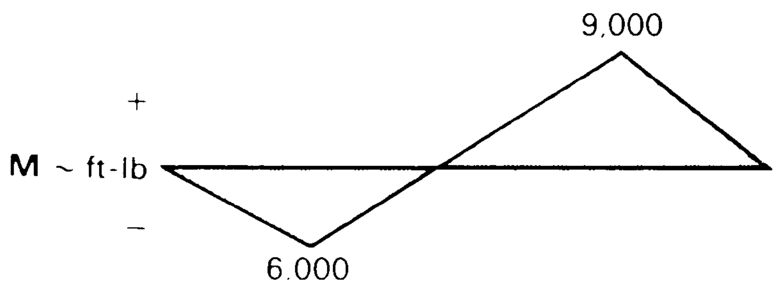
(a)



(b)



(c)



Since $\sigma = \dfrac{Mc}{I}$,

$$\sigma_{max} = \frac{9{,}000(12)5}{136} = 3{,}970 \text{ psi } T$$

and is located at the section 3 ft from the right end.

In a similar way, the maximum compressive fiber stress will occur where Mc is greatest on the compressive side of the beam. At 3 ft from the left end, the compressive stress is below the neutral axis, and

$$Mc = 6{,}000(12)5 = 360{,}000 \text{ lb-in}^2$$

At 3 ft from the right end, the compressive stress is above the neutral axis, and

$$Mc = 9{,}000(12)3 = 324{,}000 \text{ lb-in}^2$$

Therefore,

$$\sigma_{max} = \frac{6{,}000(12)5}{136} = 2{,}650 \text{ psi } C$$

and is located at the section 3 ft from the left end.

$$\epsilon_x = -y/\rho, \quad \text{where}$$

ρ = the radius of curvature of the deflected axis of the beam and

y = the distance from the neutral axis to the longitudinal fiber in question.

Using the stress-strain relationship $\sigma = E\epsilon$,

Axial Stress: $\sigma_x = -Ey/\rho$, where

σ_x = the normal stress of the fiber located y-distance from the neutral axis.

$$1/\rho = M/(EI), \quad \text{where}$$

M = the moment at the section and

I = the *moment of inertia* of the cross-section.

$$\sigma_x = -My/I, \quad \text{where}$$

y = the distance from the neutral axis to the fiber location above or below the axis. Let $y = c$, where c = distance from the neutral axis to the outermost fiber of a symmetrical beam section.

$$\sigma_x = \pm Mc/I$$

Let $S = I/c$: then

$$\sigma_x = \pm M/S, \quad \text{where}$$

S = the *elastic section modulus* of the beam member.

Beam Cross Shearing Stresses

- τ = ***VQ/Ib where***
  - ***V = Shear force from shear diagram***
  - ***Q = First moment of area above level where shear stress is desired***
  - ***I = Moment of inertia about NA***
  - ***b = thickness of beam at level where shearing stresses are desired.***

Shear Flow: $q = VQ/I$ and

Shear stress: $\tau_{xy} = VQ/(Ib)$, where

q = shear flow,

τ_{xy} = shear stress on the surface,

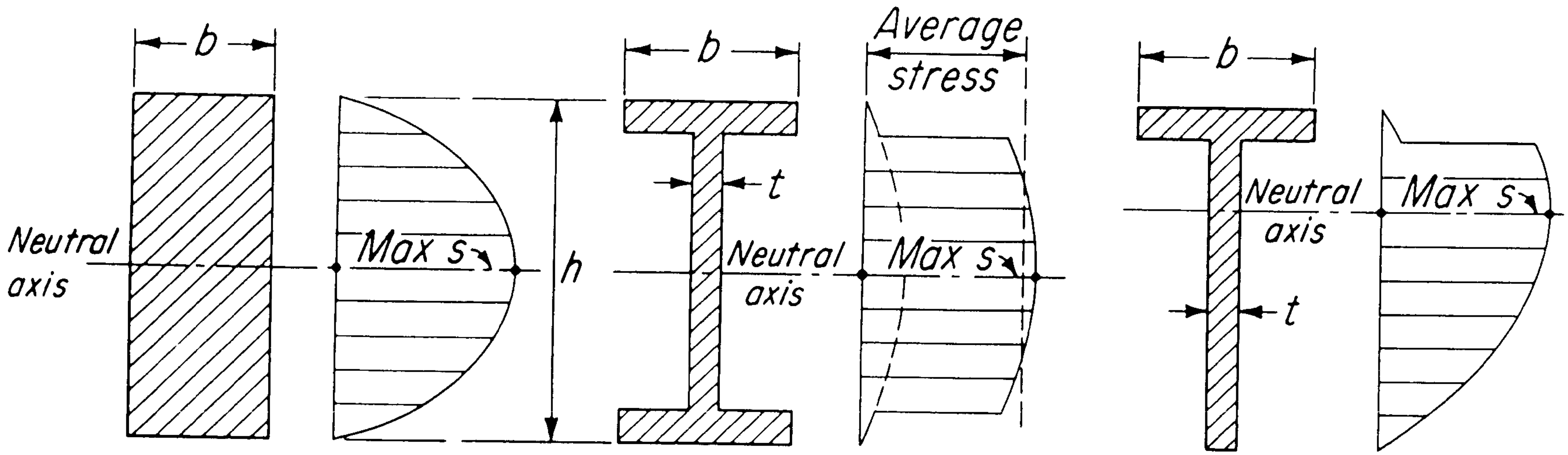
V = shear force at the section,

b = width or thickness of the cross-section, and

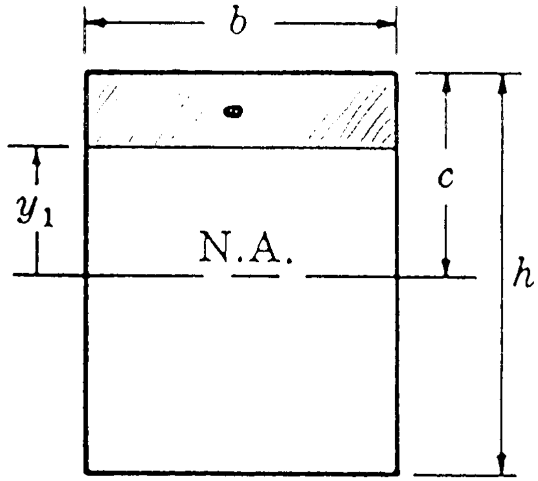
Q = $A'\bar{y}$, where

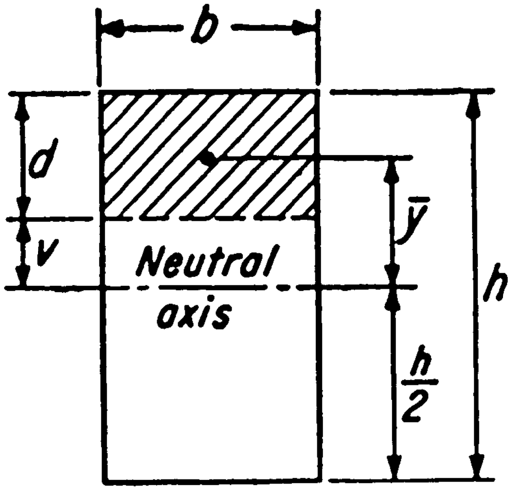
A' = area above the layer (or plane) upon which the desired shear stress acts and

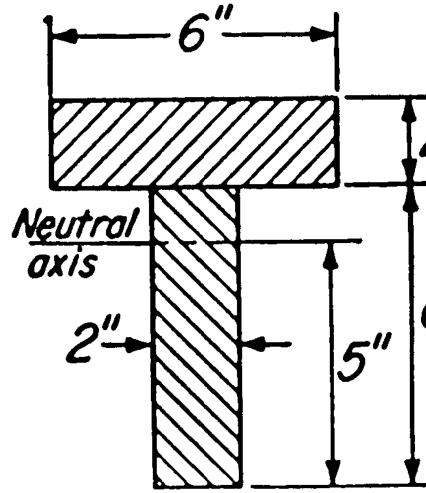
$\bar{y}$ = distance from neutral axis to area centroid.

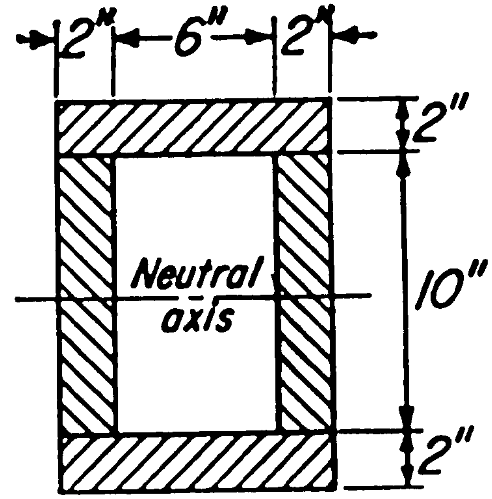

b
Neutral axis
Max s
h
b
t
Neutral axis
Average stress
Max s
b
Neutral axis
Max s
t

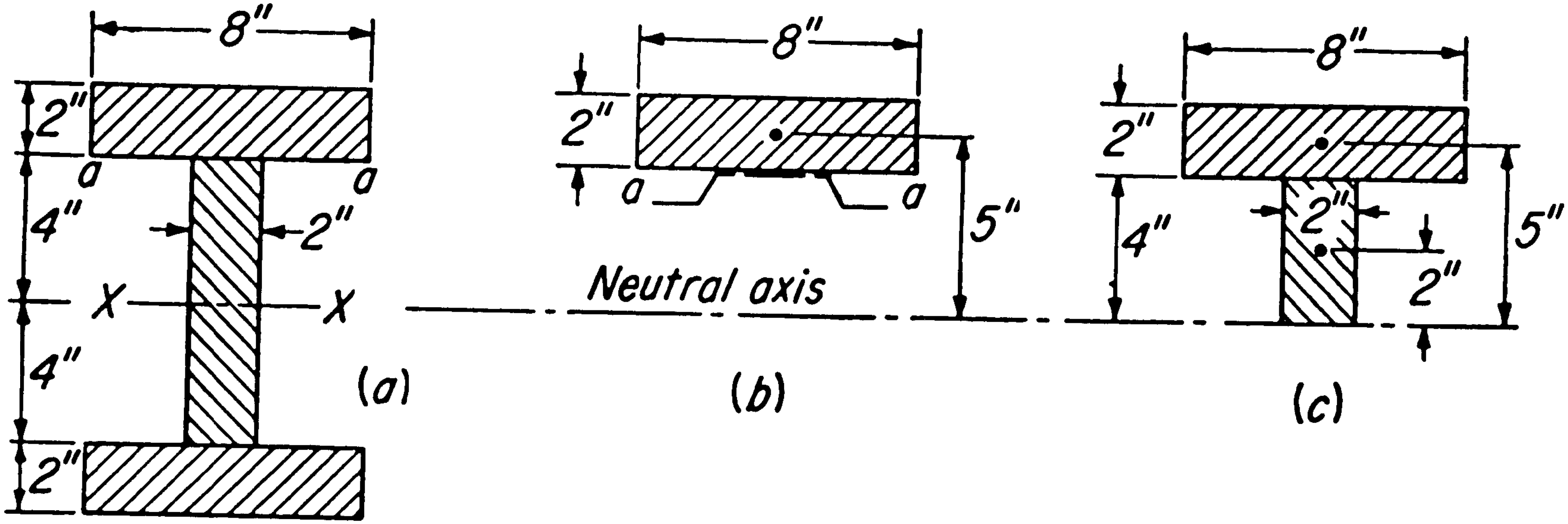
(a) (b) (c)


b
c
y_1
N.A.
h


b
d
v
$\bar{y}$
Neutral
axis
h
$\frac{h}{2}$


6"
2"
Neutral
axis
2"
5"
6"


2"
6"
2"
2"
Neutral
axis
10"
2"



$V = 4000^{\#}$

$I = 895 \text{ IN}^4$

Deflections of beams

- ***Direct integration***
- ***Tables and superposition***

Deflection of Beams

Using $1/\rho = M/(EI)$,

$$EI\frac{d^2y}{dx^2} = M, \text{ differential equation of deflection curve}$$

$$EI\frac{d^3y}{dx^3} = dM(x)/dx = V$$

$$EI\frac{d^4y}{dx^4} = dV(x)/dx = -w$$

Determine the deflection curve equation by double integration (apply boundary conditions applicable to the deflection and/or slope).

$$EI(dy/dx) = \int M(x)\,dx$$

$$EIy = \int\left[\int M(x)\,dx\right]dx$$

The constants of integration can be determined from the physical geometry of the beam.

$$M = -w(L-x)\frac{(L-x)}{2} = -\frac{w}{2}(L-x)^2,$$

$$EIv = -\frac{w}{2}\int (L-x)^2 dx = -\frac{w}{2}\left(L^2x - Lx^2 + \frac{x^3}{3}\right) + C.$$

But $v = 0$ when $x = 0$. Therefore $C = 0$, and (see Fig. 268C)

$$EIv = -\frac{w}{2}\left(L^2x - Lx^2 + \frac{x^3}{3}\right).$$

Again

$$EIy = -\frac{w}{2}\int\left(L^2x - Lx^2 + \frac{x^3}{3}\right)dx$$

$$= -\frac{w}{2}\left(\frac{L^2x^2}{2} - \frac{Lx^3}{3} + \frac{x^4}{12}\right) + C'.$$

But $y = 0$ when $x = 0$. Therefore $C' = 0$, and (see Fig. 268D)

$$y = -\frac{w}{2EI}\left(\frac{L^2x^2}{2} - \frac{Lx^3}{3} + \frac{x^4}{12}\right).$$

The greatest slope and deflection occur at the end of the beam where $x = L$; their values are

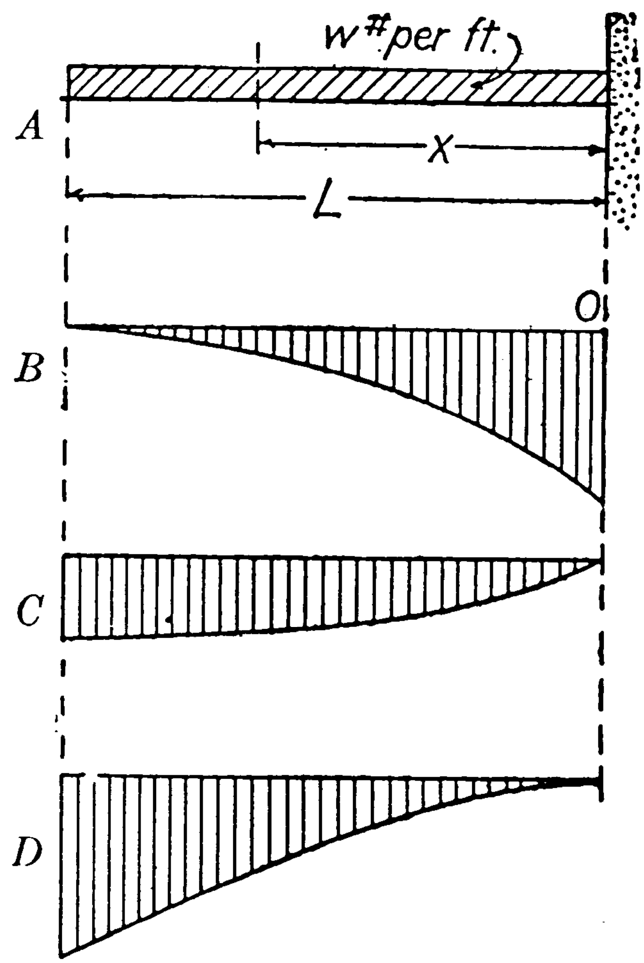
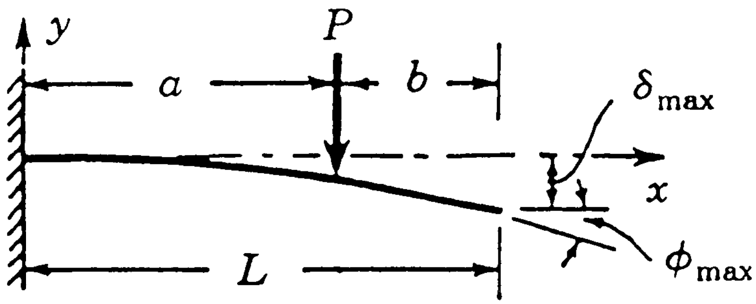


FIG. 268

$$v \max = -\frac{wL^3}{6EI} = -\frac{WL^2}{6EI},$$

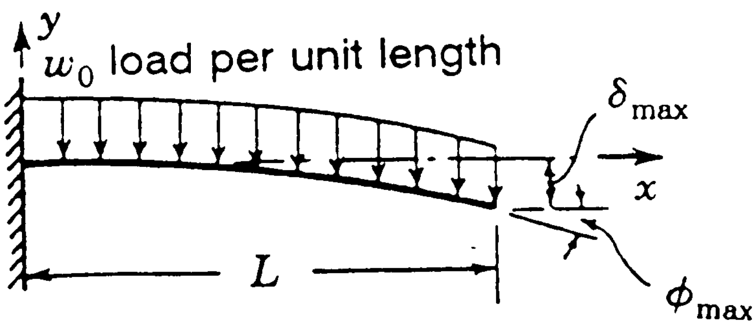
$$y \max = -\frac{wL^4}{8EI} = -\frac{WL^3}{8EI}.$$

Beam Deflection Formu
(δ is positive d

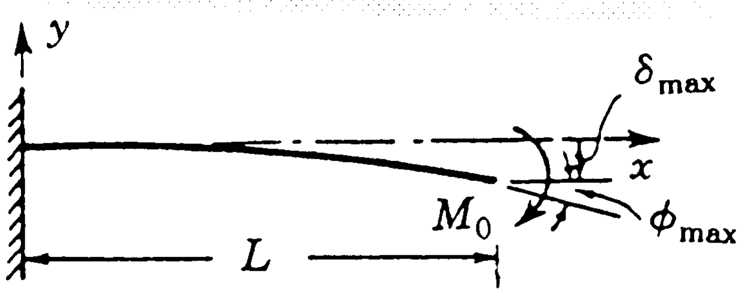


$$\delta = \frac{Pa^2}{6EI}(3x - a), \text{ for } x > a$$

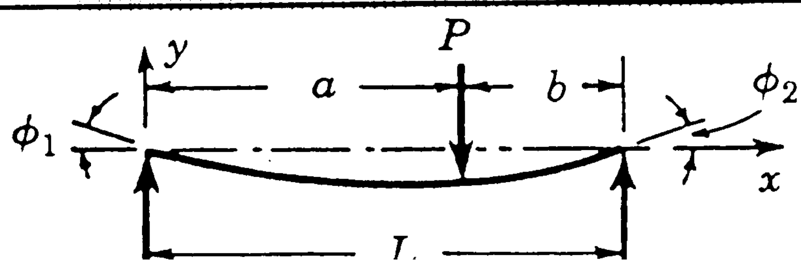
$$\delta = \frac{Px^2}{6EI}(-x + 3a), \text{ for } x \le a$$



$$\delta = \frac{w_o x^2}{24EI}(x^2 + 6L^2 - 4Lx)$$



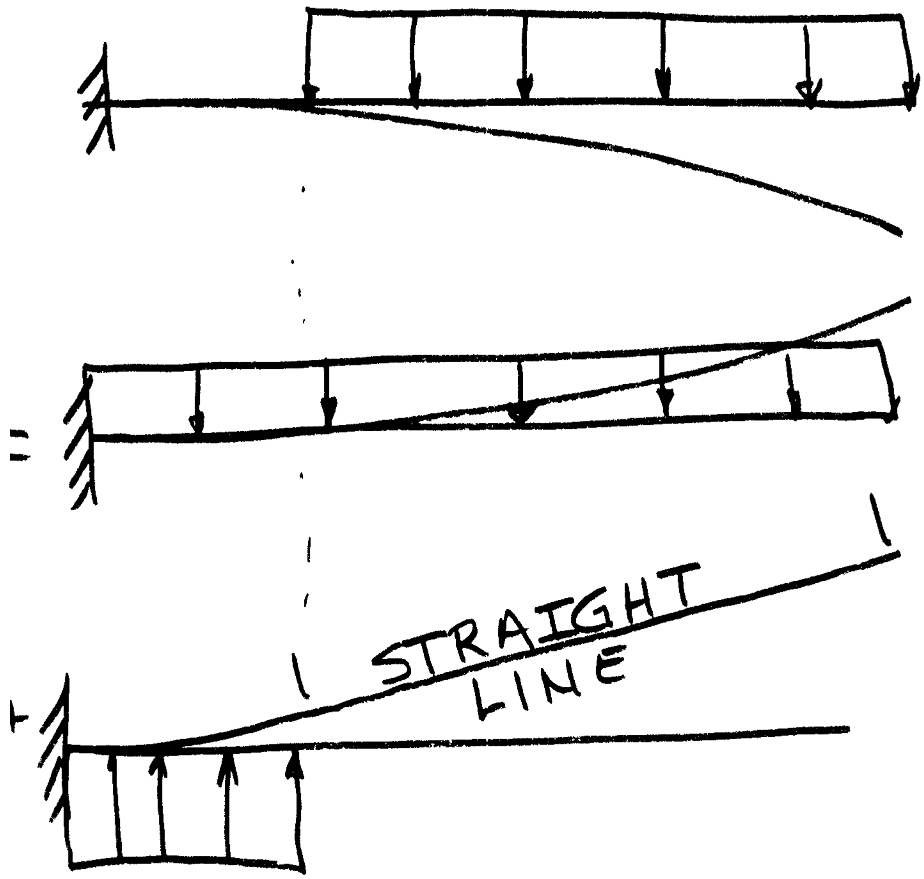
$$\delta = \frac{M_o x^2}{2EI}$$



$$\delta = \frac{Pb}{6LEI}\left[\frac{L}{b}(x-a)^3 - x^3 + (L^2 - b^2)x\right]$$

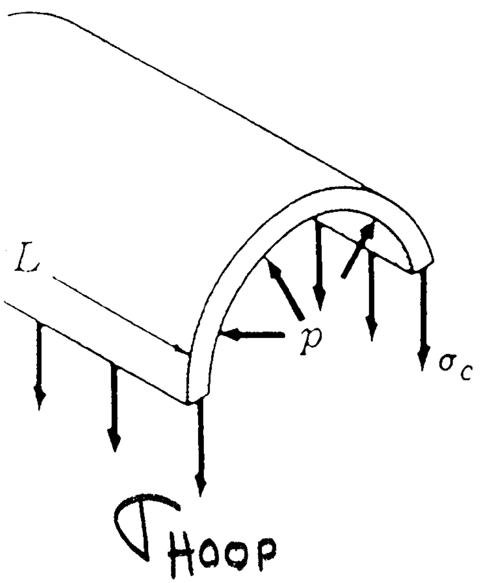
$$\delta = \frac{Pb}{6LEI}[-x^3 + (L^2 - b^2)x], \text{ for } x \le$$

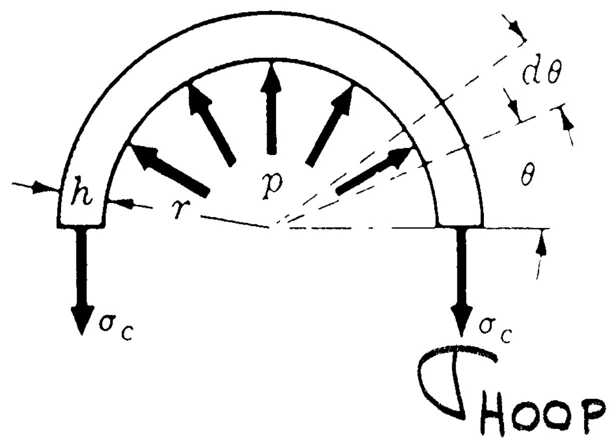
Pg 33
REF MANUAL

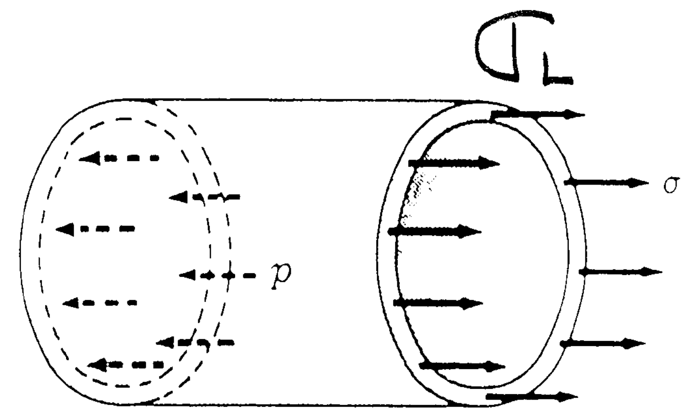


Cylindrical Pressure Vessels

- ***Hoop stresses act around the tank, in tension -*** $\sigma_H = pD/2t$
- ***Longitudinal or axial stresses act along the axis of the tank -*** $\sigma_L = pD/4t$
- ***Maximum shear stresses -*** $\tau = pD/4t$


L
p
σ_c
σ_{HOOP}


h
r
p
$d\theta$
θ
σ_c
σ_c
σ_{HOOP}


σ_L
σ
p

Spherical Pressure Vessels

- ***All tensile stresses in all directions***

  $\sigma_H = pr/2t$

- ***Max. shear stresses -*** $\tau = pr/4t$

THIN WALLED CYLINDRICAL PRESSURE VESSEL

Hoop Tension

$$\sigma_t = pD/2t, \quad \text{where}$$

σ_t = hoop stress,
p = the uniform internal pressure,
D = internal diameter of cylinder, and
t = wall thickness.

Axial Tension

$$\sigma_a = pD/4t, \quad \text{where}$$

σ_a = axial stress of tank.

Columns

- ***Pinned on each end - $Pcr = \pi^2 EI / (KL)^2$***
  - ***Pcr is the critical failure buckling load with no factor of safety,***
  - ***E is the modulus of elasticity,***
  - ***I is the moment about the weak or buckling axis of the column,***
  - ***L is the length between points of zero moment or points of inflection,***
  - ***K is the effective length factor.***

Columns acting as beams

- ***Pinned on each end, eccentrically loaded short columns too short to buckle***

  = F/A +- Mc/I where F/A is the regular axial stress, and Mc/I is the regular bending stress.

COLUMNS

Beam-Columns (axially-loaded beams)

Combining Stresses (eccentrically-loaded short columns)

$$\sigma_{\max}, \sigma_{\min} = F/A \pm Mc/I$$

Long Columns--Euler's Formula

$$P_{cr} = \pi^2 EI/(kl)^2, \text{ where}$$

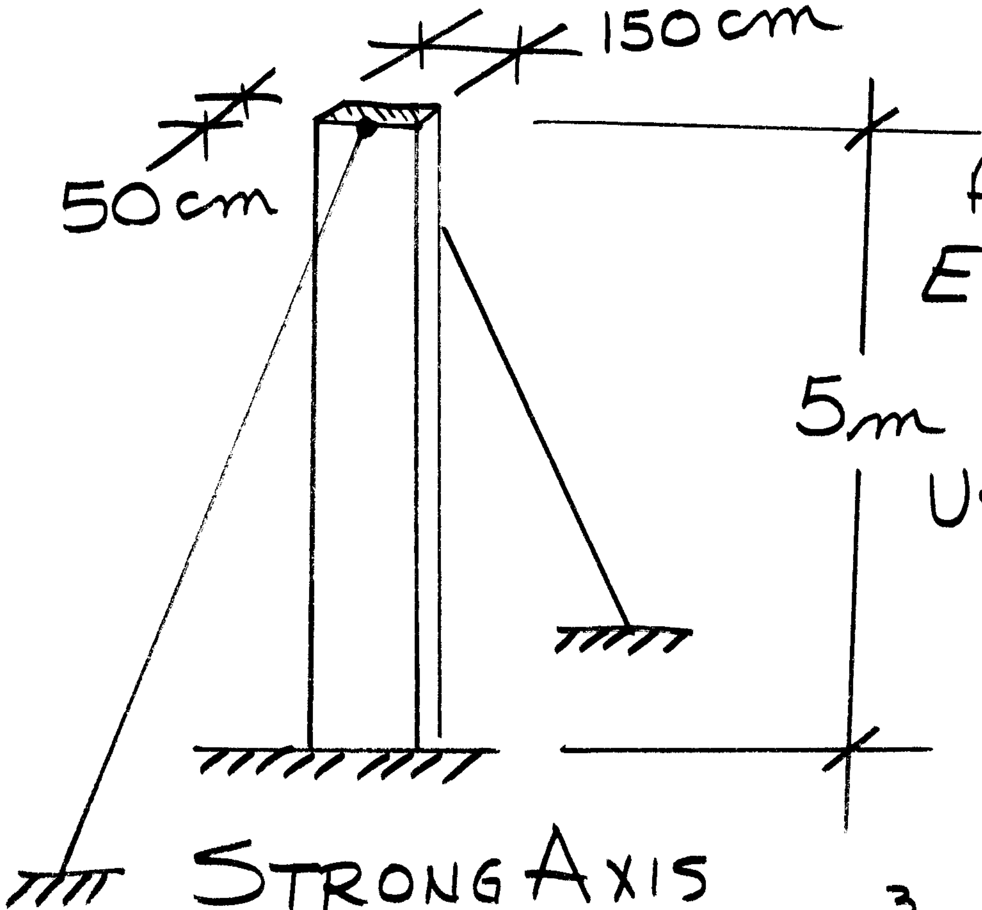
P_{cr} = critical axial loading,
k = a constant determined by column end restraints, and
l = unbraced column length.

Substitute $I = r^2 A$:

$$P_{cr}/A = \pi^2 E/[k(l/r)]^2, \text{ where}$$

r = *radius of gyration* and
l/r = *slenderness ratio* for the column.

Commonly Used k Values For Columns		
Theoretical Value	**Design Value**	**End Condition**
0.5	0.65	both ends fixed
0.7	0.80	one end fixed and other end pinned
1.0	1.00	both ends pinned
2.0	2.10	one end fixed and



ALUMINUM

$E = 6.9 \times 10^{10}$ Pa

(N/m^2)

USE <u>DESIGN</u>

<u>STRONG AXIS</u>

$I = (0.050)(0.150)^3/12 = 14.06 \times 10^{-6}\,m^4$

$k = 2.1$

<u>WEAK AXIS</u>

$I = (0.150)\,(0.050)^3/12 = 1.56 \times 10^{-6}\,m^4$

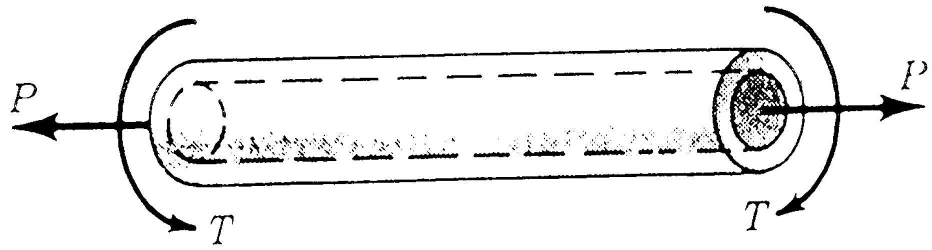
$k = 0.8$

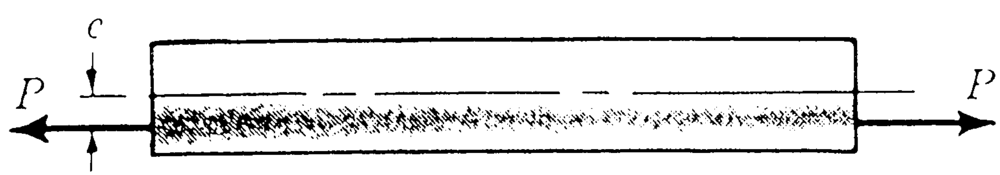
Combined Stresses

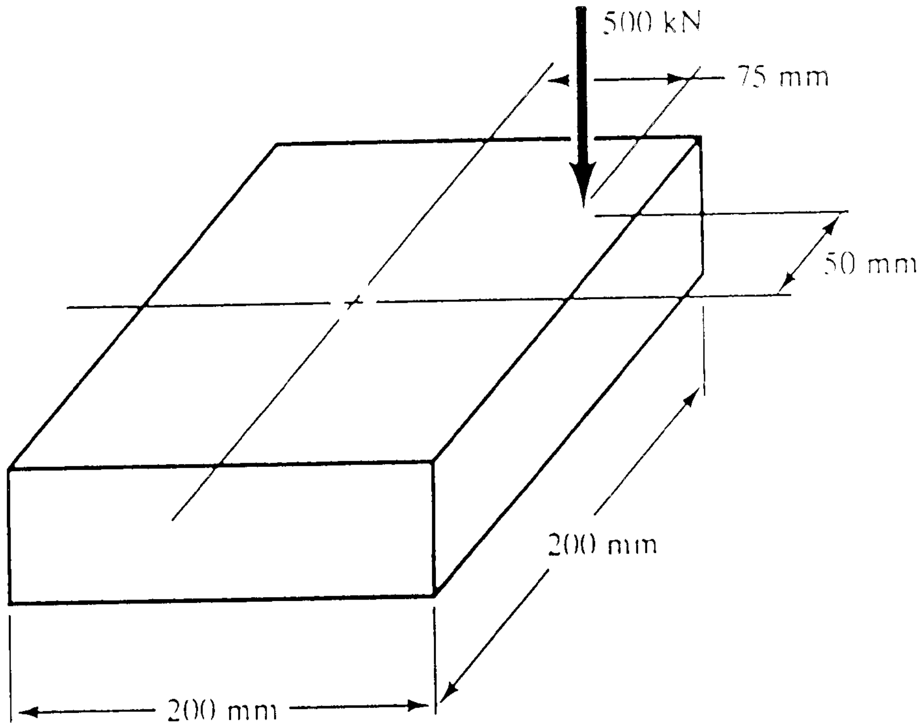
- ***Axial and torsion (tension and shear)***
- ***Axial and bending (tension and tension)***
- ***Stresses under footings (compression and compression)***
- ***Axial and torsion and bending and pressure (cheee!)***
- ***Plane stress equations for combined stresses***

Combined Stresses

- ***Compute axial stresses and torsional stresses and bending stresses and hoop and longitudinal pressure stresses separately and put them on the stress block. Then determine principal stresses from principal stress equations (see later.)***

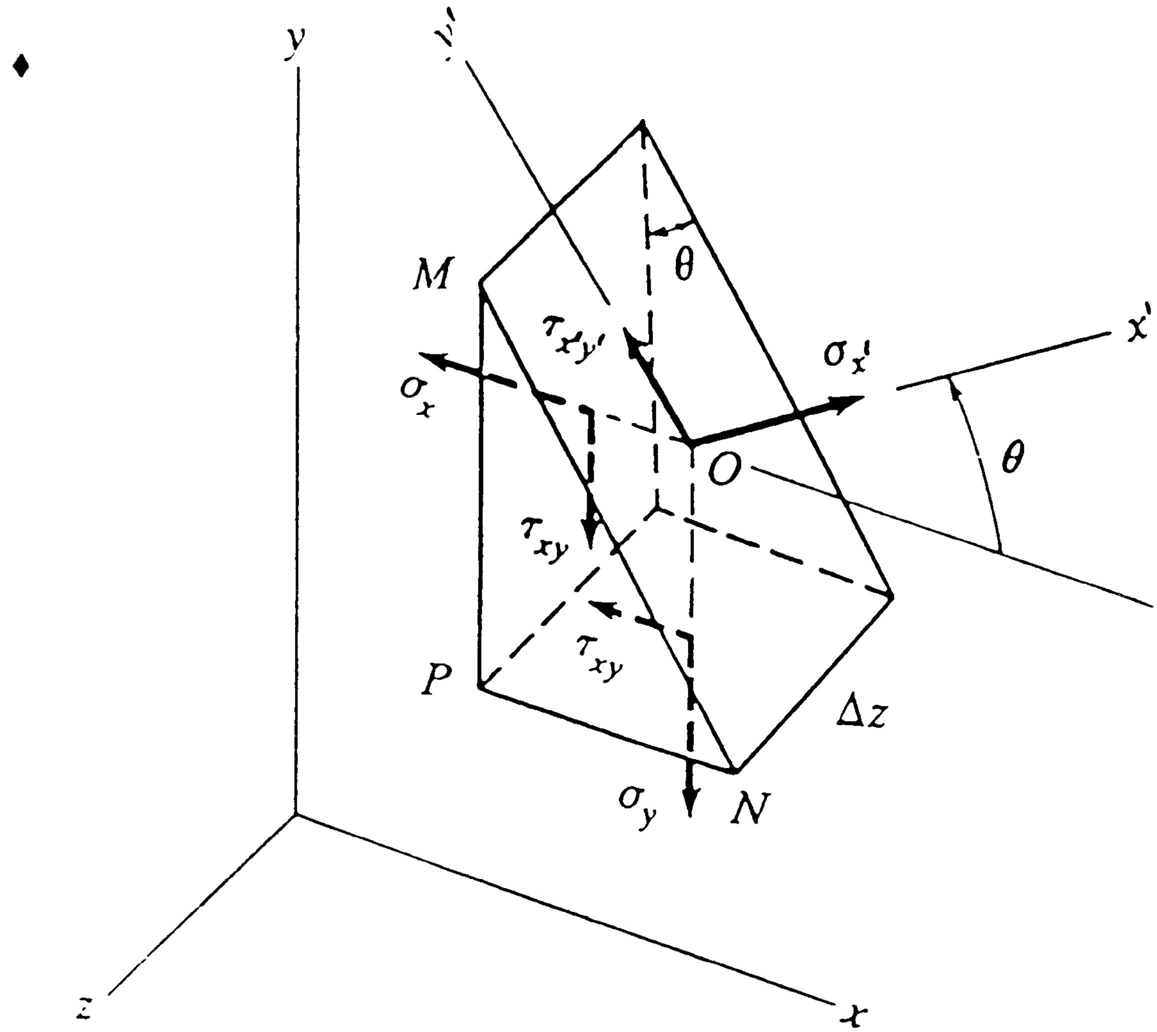

P
P
T
T


c
P
P


500 kN
75 mm
50 mm
200 mm
200 mm

Stresses on an plane and Principal Stresses

- ***Knowing σ_x, σ_y, and τ_{xy}, use the stress transformation equations (From Statics) to find stresses on any other plane.***
- ***Knowing σ_x, σ_y, and τ_{xy}, use Mohr's circle to determine the principal stresses.***



From Statics:

$$\sigma_{x'} = (\sigma_x + \sigma_y)/2 + [(\sigma_x - \sigma_y)/2]\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = (\sigma_x + \sigma_y)/2 - [(\sigma_x - \sigma_y)/2]\cos 2\theta - \tau_{xy}\sin 2\theta$$

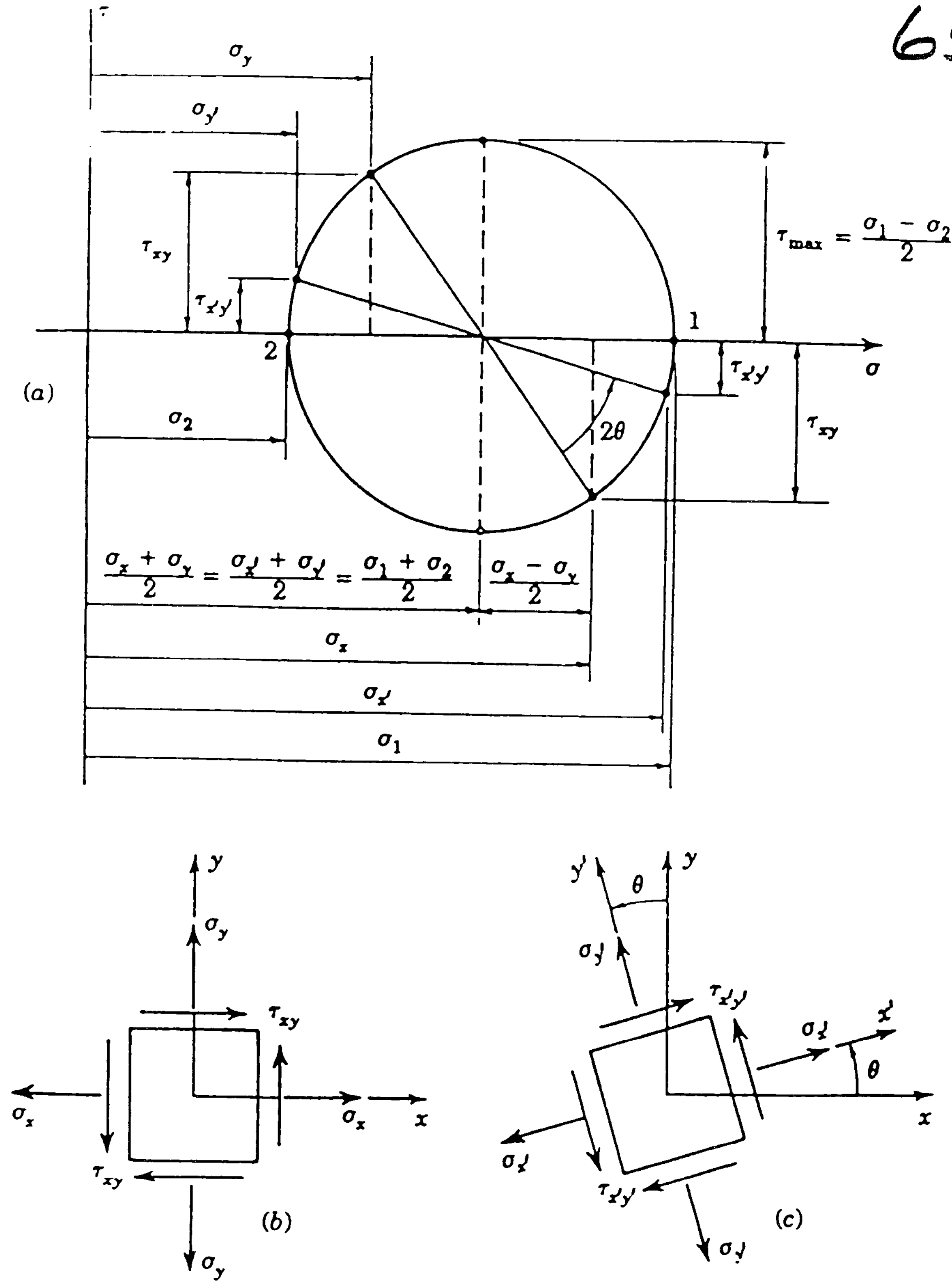
$$\tau_{x'y'} = -[(\sigma_x - \sigma_y)/2]\sin 2\theta + \tau_{xy}\cos 2\theta$$

Mohr's Circle--Stress

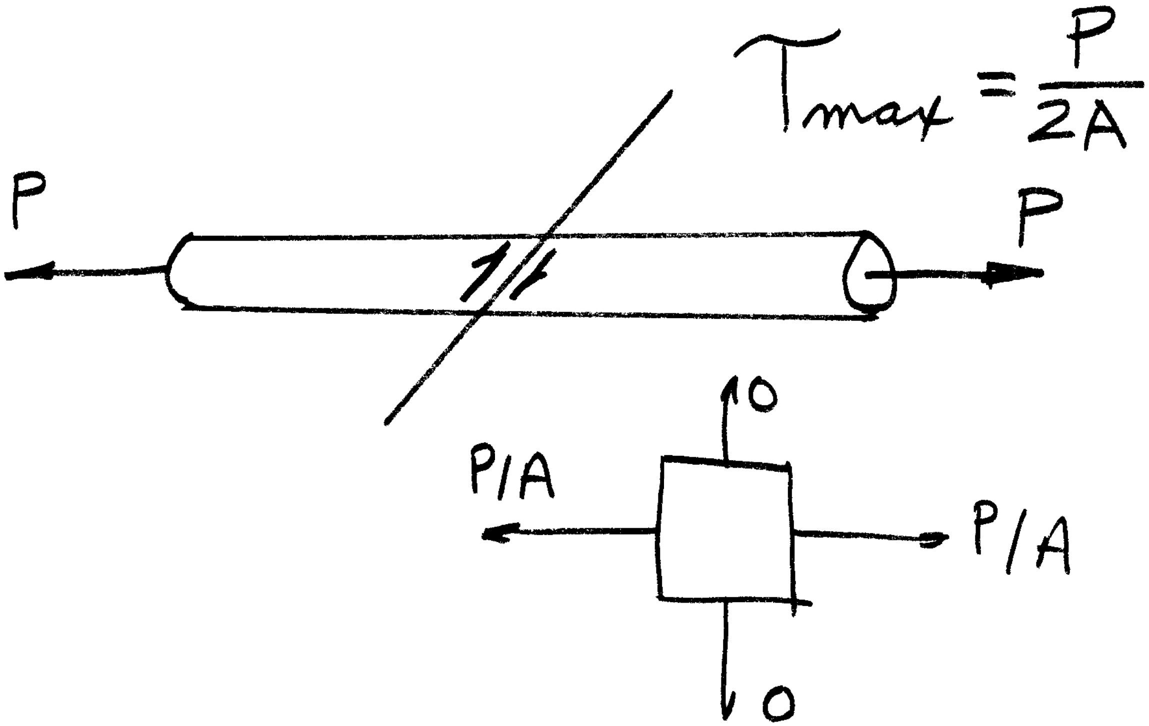
The stresses on a specified plane surface can be determined from the stresses on two other surfaces which are perpendicular to each other.

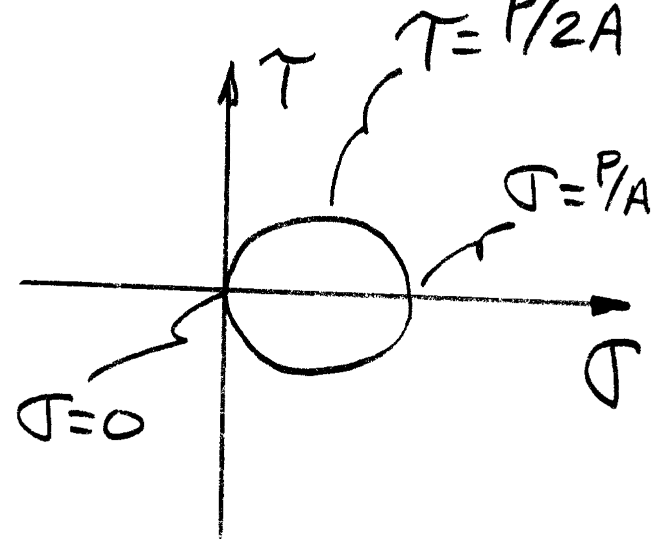
To construct a *Mohr's circle*, the following sign conventions are used.

1. Tensile normal stress components are considered positive. Compressive normal stress components are negative.
2. Shearing stresses will be considered positive when the pair of shearing stresses, acting on opposite and parallel faces of an element, forms a clockwise couple. Negative, a counterclockwise couple.

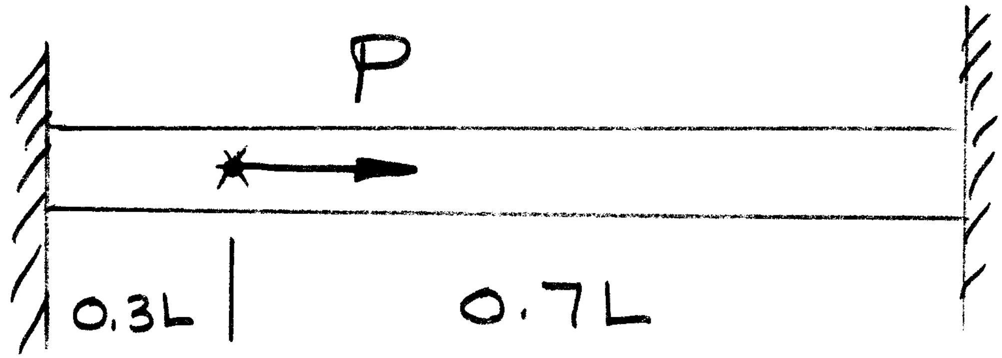

τ
σ_y
$\sigma_{y'}$
τ_{xy}
$\tau_{x'y'}$
$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$
1
2
σ
(a)
σ_2
2θ
$\frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_{x'} + \sigma_{y'}}{2} = \frac{\sigma_1 + \sigma_2}{2}$
$\frac{\sigma_x - \sigma_y}{2}$
σ_x
$\sigma_{x'}$
σ_1
y
σ_y
τ_{xy}
σ_x
x
(b)
y'
θ
$\sigma_{y'}$
$\tau_{x'y'}$
$\sigma_{x'}$
x'
(c)

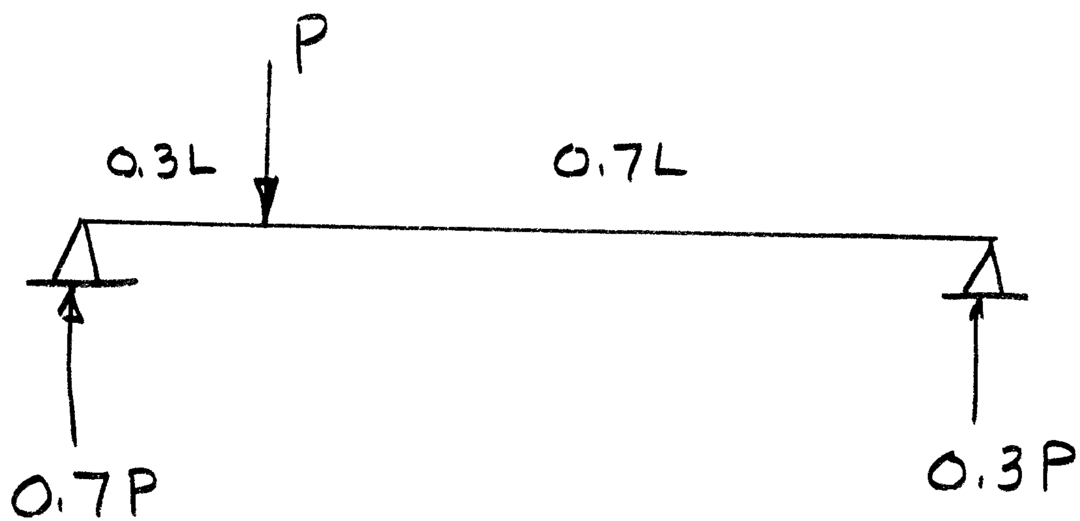
Shear stresses on bars subjected to pure tension or compression



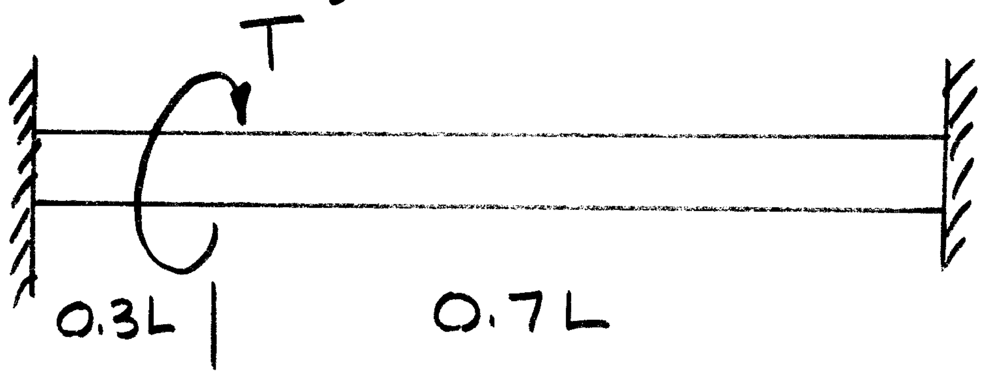


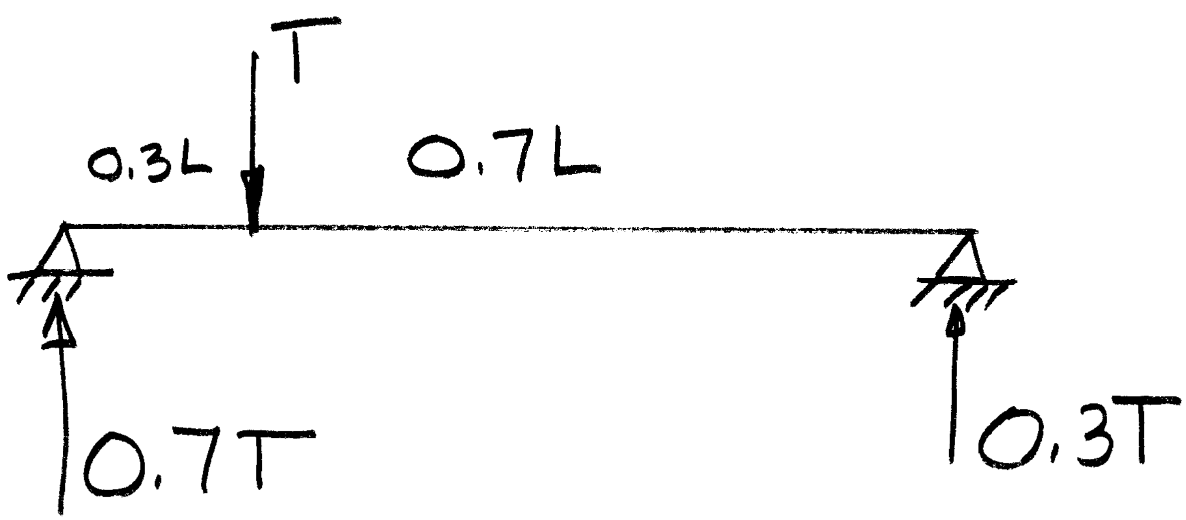
Loads in statically indeterminate axially loaded bars



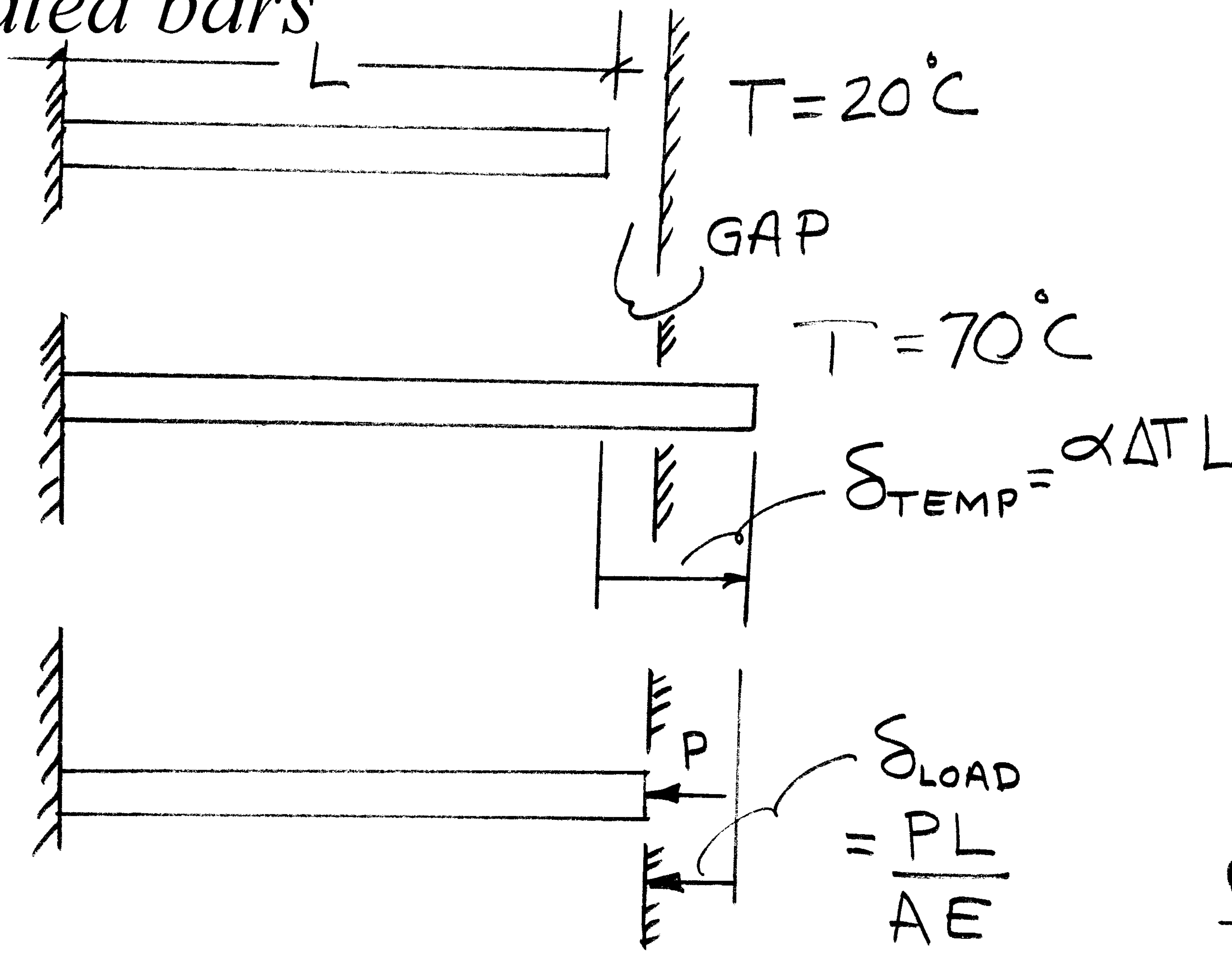


Loads in statically indeterminate torsionally loaded bars

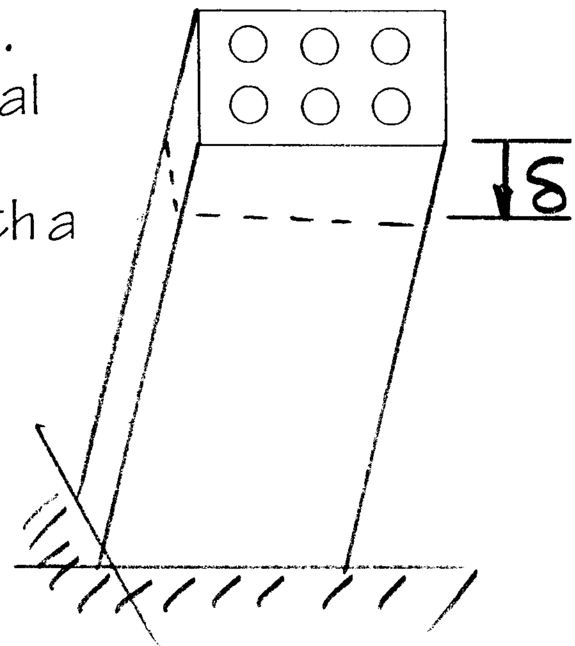




Loads in statically indeterminate heated bars



Determine the stress induced in the concrete and in the steel bars shown. Use 6 each 2 inch diameter structural steel bars and fairly high strength concrete. The column is 10" by 18" with a load of 1800 kips.



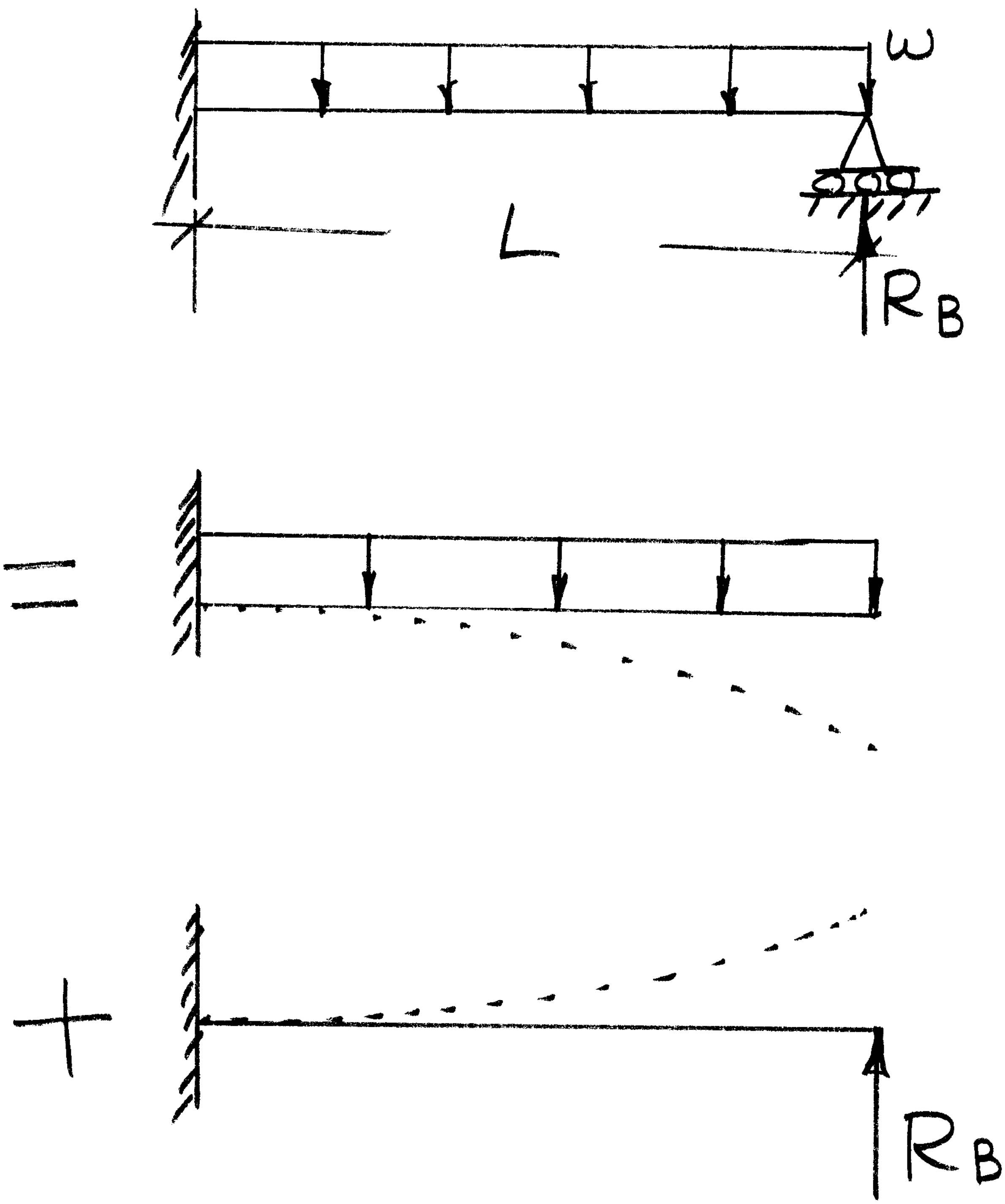
$$\delta_{CONCRETE} = \delta_{STEEL}$$

1) $$\frac{P_c L_c}{A_c E_c} = \frac{P_S L_S}{A_S E_S}$$

2) $$P_{TOTAL} = P_C + P_S$$

SEE PG 33 - REF. MAN.

Loads in statically indeterminate beams



Maximum-Normal-Stress Theory

The maximum-normal-stress theory states that failure occurs when one of the three principal stresses equals the strength of the material. If $\sigma_1 > \sigma_2 > \sigma_3$ then the theory predicts that failure occurs whenever $\sigma_1 \geq S_t$ or $\sigma_3 \leq -S_c$ where S_t and S_c are the tensile and compressive strengths, respectively.

Maximum-Shear-Stress Theory

The maximum-shear-stress theory states that yielding begins when the maximum shear stress equals the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield. If $\sigma_1 > \sigma_2 > \sigma_3$ then the theory predicts that yielding will occur whenever $\tau_{max} \geq S_y/2$ where S_y is the yield strength.

Distortion-Energy Theory

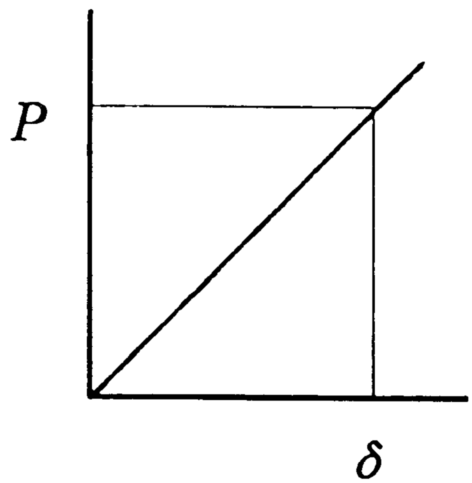
The distortion-energy theory states that yielding begins whenever the distortion energy in a unit volume equals the distortion energy in the same volume when uniaxially stressed to the yield strength. The theory predicts that yielding will occur whenever
$\{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2]/2\}^{1/2} \geq S_y$.

ELASTIC STRAIN ENERGY

If the strain remains within the elastic limit, the work done during deflection (extension) of a member will be transformed into potential energy and can be recovered.

If the final load is P and the corresponding elongation of a tension member is δ, then the total energy U stored is equal to the work W done during loading.

$$U = W = P\delta/2$$



The strain energy per unit volume is

$$u = U/AL = \sigma^2/2E \qquad \text{(for tension)}$$