

STATICS

Fundamentals of Engineering Exam Review

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FORCE

A *force* is a *vector* quantity. It is defined when its (1) magnitude, (2) point of application, and (3) direction are known.

RESULTANT (Two Dimensions)

The *resultant*, F , of n forces with components $F_{x,i}$ and $F_{y,i}$ has the magnitude of

$$F = \left[\left(\sum_{i=1}^n F_{x,i} \right)^2 + \left(\sum_{i=1}^n F_{y,i} \right)^2 \right]^{1/2}$$

The resultant direction with respect to the x -axis using four-quadrant angle functions is

$$\theta = \arctan \left(\sum_{i=1}^n F_{y,i} / \sum_{i=1}^n F_{x,i} \right)$$

The vector form of the force is

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}$$

RESOLUTION OF A FORCE

$$F_x = F \cos \theta_x; F_y = F \cos \theta_y; F_z = F \cos \theta_z$$

$$\cos \theta_x = F_x/F; \cos \theta_y = F_y/F; \cos \theta_z = F_z/F$$

Separating a force into components (geometry of force is known --- $R = \sqrt{x^2 + y^2 + z^2}$)

$$F_x = (x/R)F; F_y = (y/R)F; F_z = (z/R)F$$

MOMENTS (COUPLES)

A system of two forces that are equal in magnitude, opposite in direction, and parallel to each other is called a *couple*.

A *moment* M is defined as the cross product of the *radius vector* distance r and the *force* F from a point

$$F_x = (x/R)F; \quad F_y = (y/R)F; \quad F_z = (z/R)F$$

MOMENTS (COUPLES)

A system of two forces that are equal in magnitude, opposite in direction, and parallel to each other is called a *couple*.

A *moment M* is defined as the cross product of the *radius vector* distance *r* and the *force F* from a point to the line of action of the force.

$$\begin{aligned} M &= r \times F; & M_x &= yF_z - zF_y, \\ & & M_y &= zF_x - xF_z, \text{ and} \\ & & M_z &= xF_y - yF_x. \end{aligned}$$

SYSTEMS OF FORCES

$$\begin{aligned} F &= \Sigma F_n \\ M &= \Sigma (r_n \times F_n) \end{aligned}$$

Equilibrium Requirements

$$\begin{aligned} \Sigma F_n &= 0 \\ \Sigma M_n &= 0 \end{aligned}$$

CENTROIDS OF MASSES, AREAS, LENGTHS, AND VOLUMES

Formulas for centroids, moments of inertia, and first moment of areas are presented in the **MATHEMATICS** section for continuous functions. The following discrete formulas are for defined regular masses, areas, lengths, and volumes:

$$r_c = \Sigma m \cdot r / \Sigma m \quad \text{where}$$

m_n = the mass of each particle making up the system,

r_n = the radius vector to each particle from a selected reference point, and

r_c = the radius vector to the center of the total mass from the selected reference point.

The *moment of area* (M_a) is defined as

$$M_{ay} = \Sigma x_n a_n, \text{ with respect to the } y\text{-axis.}$$

$$M_{ax} = \Sigma y_n a_n, \text{ with respect to the } x\text{-axis.}$$

The *centroid of area* is defined as

$$\left. \begin{aligned} x_{ac} &= M_{ay}/A \\ y_{ac} &= M_{ax}/A \\ z_{ac} &= M_{az}/A \end{aligned} \right\} \text{ with respect to center} \\ \text{of the coordinate system}$$

where $A = \Sigma a_n$

The *centroid of a line* is defined as

$$x_{lc} = (\Sigma x_n l_n)/L, \text{ where } L = \Sigma l_n$$

$$y_{lc} = (\Sigma y_n l_n)/L$$

$$z_{lc} = (\Sigma z_n l_n)/L$$

The *centroid of volume* is defined as

$$x_{vc} = (\Sigma x_n v_n)/V, \text{ where } V = \Sigma v_n$$

$$y_{vc} = (\Sigma y_n v_n)/V$$

$$z_{vc} = (\Sigma z_n v_n)/V$$

MOMENT OF INERTIA

The *moment of inertia*, or the second moment of area, is defined as

$$I_x = \int x^2 dA$$

$$x_{vc} = (\Sigma x_n v_n)/V, \text{ where } V = \Sigma v_n$$

$$y_{vc} = (\Sigma y_n v_n)/V$$

$$z_{vc} = (\Sigma z_n v_n)/V$$

MOMENT OF INERTIA

The *moment of inertia*, or the second moment of area, is defined as

$$I_y = \int x^2 dA$$

$$I_x = \int y^2 dA$$

The *polar moment of inertia* J of an area about a point is equal to the sum of the moments of inertia of the area about any two perpendicular axes in the area and passing through the same point.

$$\begin{aligned} I_z = J &= I_{y_c} + I_{x_c} = \int (x^2 + y^2) dA \\ &= r_p^2 A, \text{ where} \end{aligned}$$

r_p = the *radius of gyration* (see next page).

Moment of Inertia Transfer Theorem

The moment of inertia of an area about any axis is defined as the moment of inertia of the area about a parallel centroidal axis plus a term equal to the area multiplied by the square of the distance d from the centroidal axis to the axis in question.

$$I'_x = I_{x_c} + d^2 A$$

$$I'_y = I_{y_c} + d^2 A, \text{ where}$$

d = distance between the two axes in question,

I_{x_c}, I_{y_c} = the moment of inertia about the centroidal axis and

I'_x, I'_y = the moment of inertia about the new axis. 5

Radius of Gyration

The *radius of gyration* r_p, r_x, r_y is the distance from a reference axis at which all of the area can be considered to be concentrated to produce the moment of inertia.

$$r_x = \sqrt{I_x/A} ; \quad r_y = \sqrt{I_y/A} ; \quad r_p = \sqrt{J/A}$$

Product of Inertia

The *product of inertia* (I_{xy} , etc.) is defined as:

$$I_{xy} = \int xy dA, \text{ with respect to the } xy\text{-coordinate system,}$$

$$I_{xz} = \int xz dA, \text{ with respect to the } xz\text{-coordinate system, and}$$

$$I_{yz} = \int yz dA, \text{ with respect to the } yz\text{-coordinate system.}$$

The *transfer theorem* also applies:

$$I'_{xy} = I_{x_c y_c} + d_x d_y A, \quad \text{for the } xy\text{-coordinate system, etc., where}$$

$$d_x = x\text{-axis distance between the two axes in question and}$$

$$d_y = y\text{-axis distance between the two axes in question.}$$

FRICITION

The largest frictional force that is possible to develop is called the *limiting friction*. Any further increase in applied forces would cause motion.

$$F = \mu N, \text{ where}$$

tion and
 d_y = y -axis distance between the two axes in question.

FRICTION

The largest frictional force that is possible to develop is called the *limiting friction*. Any further increase in applied forces would cause motion.

$$F = \mu N, \text{ where}$$

F = friction force,

μ = *coefficient of static friction*, and

N = normal force between surfaces in contact.

SCREW THREAD

For a *screw-jack, square thread*,

$$M = Pr \tan (\alpha \pm \phi), \text{ where}$$

+ is for screw tightening,

- is for screw loosening,

M = external moment applied to axis of screw,

P = load on jack applied along and on the line of the axis,

r = the mean thread radius,

α = the *pitch angle* of the thread, and

$\mu = \tan \phi =$ the appropriate coefficient of friction.

BRAKE-BAND OR BELT FRICTION

$$F_1 = F_2 e^{\mu \theta}, \text{ where}$$

F_1 = force being applied in the direction of impending motion

- 7
- r_2 = force applied to resist impending motion,
 μ = coefficient of static friction, and
 θ = the total *angle of contact* between the surfaces expressed in radians.

STATICALLY DETERMINATE TRUSS

Plane Truss

A plane truss is a rigid framework satisfying the following conditions:

1. The members of the framework lie in the same plane.
2. The members are connected at their ends by frictionless pins.
3. All of the external loads lie in the plane of the framework and are applied at the joints only.
4. The truss reactions and member forces can be determined using the equations of equilibrium.

$$\Sigma F = 0; \quad \Sigma M = 0$$

5. A truss is statically indeterminate if the reactions and member forces cannot be solved with the equations of equilibrium.

Plane Truss: Method of Joints

The method consists of solving for the forces in the members by writing the two equilibrium equations for each joint of the truss.

$$\Sigma F_V = 0 \quad \text{and} \quad \Sigma F_H = 0, \quad \text{where}$$

- F_H = horizontal forces and member components and
 F_V = vertical forces and member components.

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- F_H = horizontal forces and member components and
- F_V = vertical forces and member components.

Plane Truss: Method of Sections

The method consists of drawing a free-body diagram of a portion of the truss in such a way that the desired truss member unknown force is exposed as an external force.

CONCURRENT FORCES

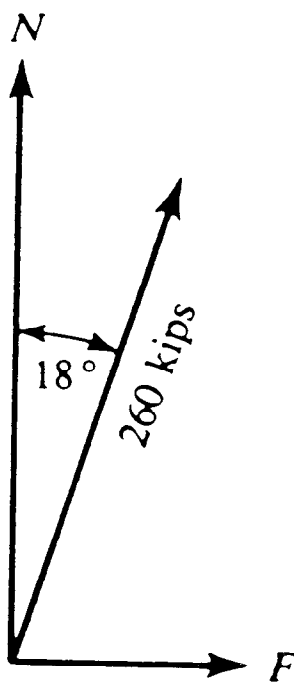
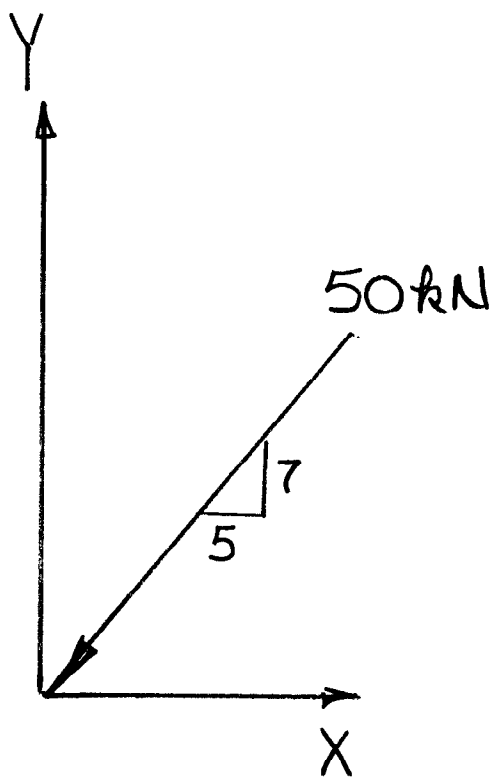
A system of forces wherein their lines of action all meet at one point.

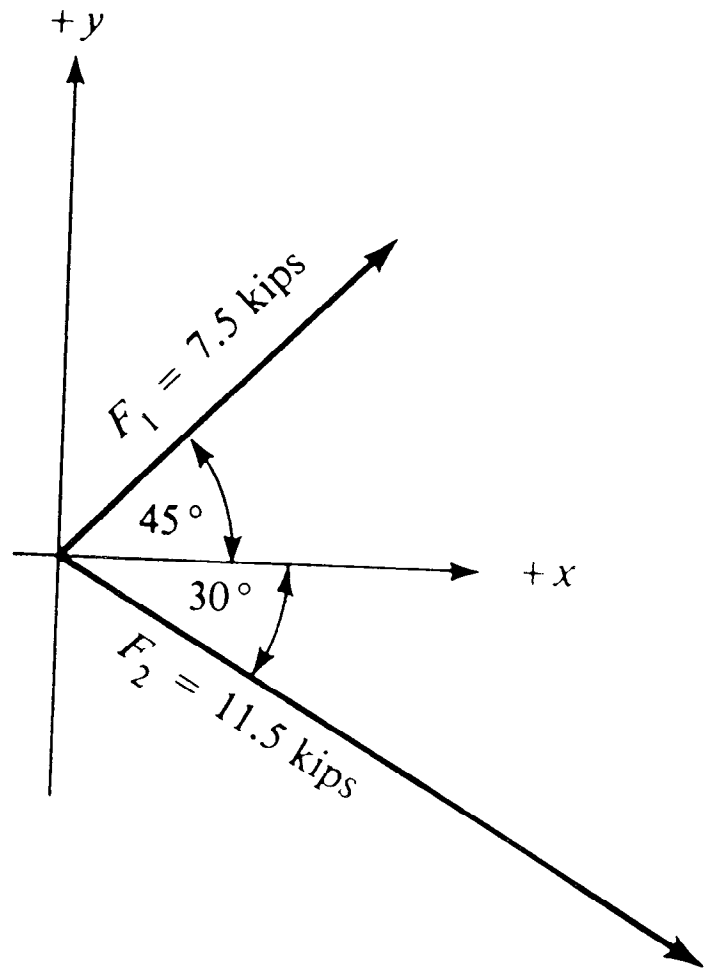
Two Dimensions

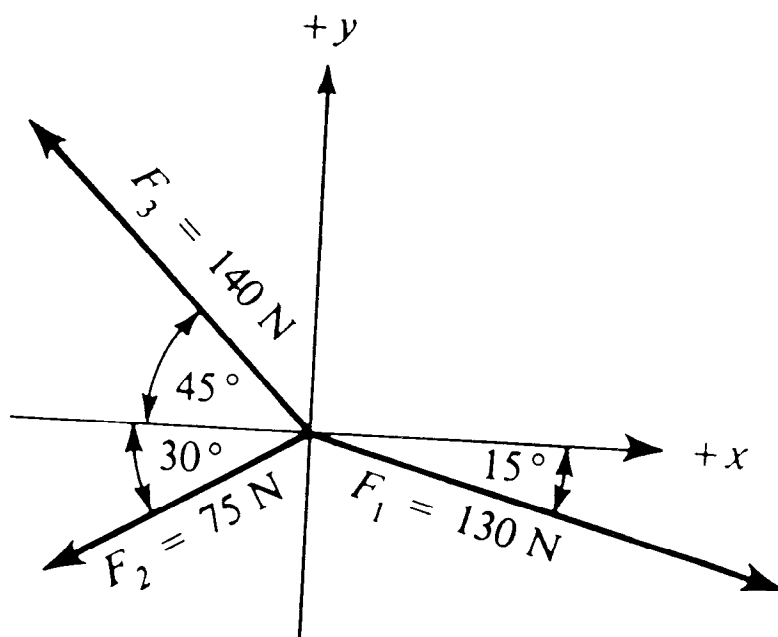
$$\Sigma F_x = 0; \quad \Sigma F_y = 0$$

Three Dimensions

$$\Sigma F_x = 0; \quad \Sigma F_y = 0; \quad \Sigma F_z = 0$$

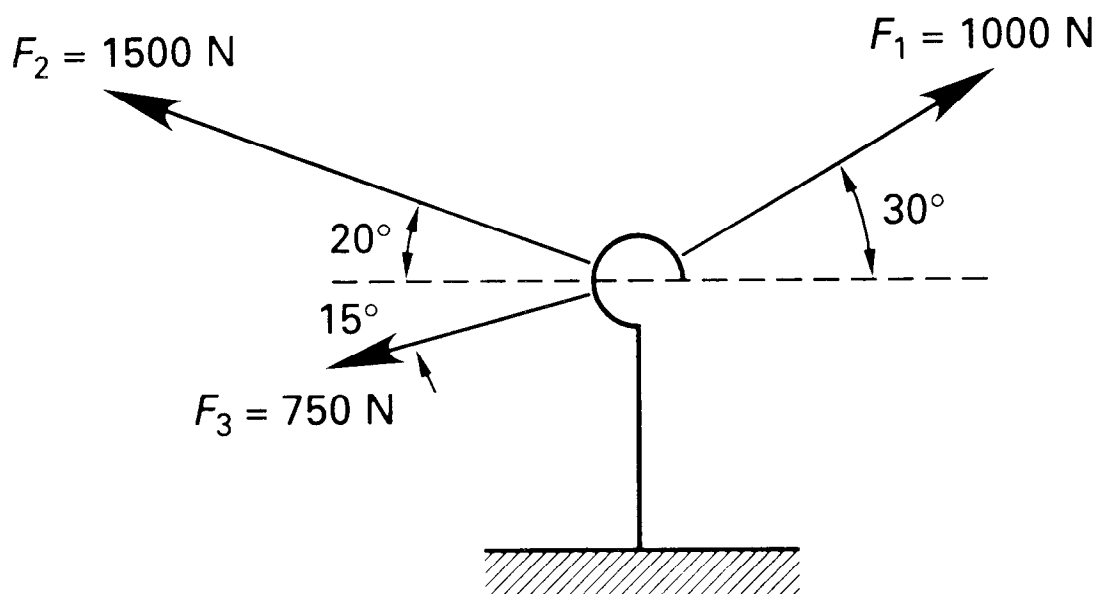






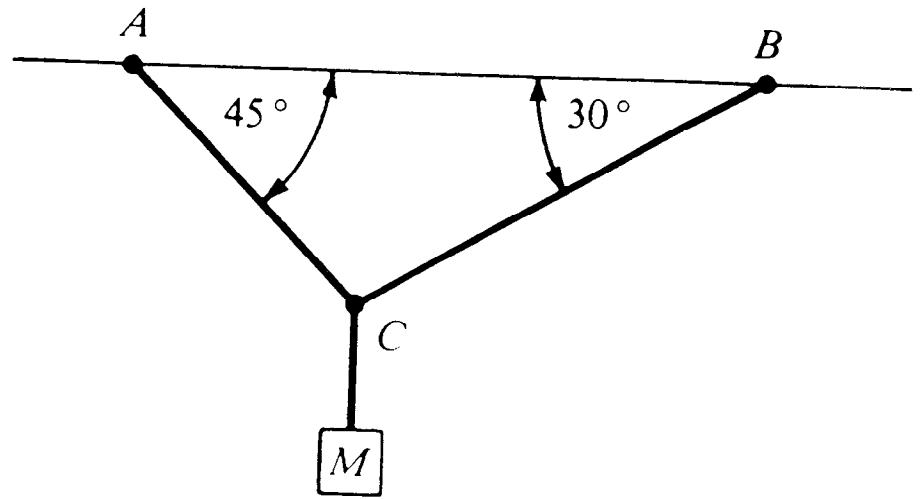
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4. Three forces act on a hook. Determine the magnitude of the resultant of the forces. Neglect hook bending.



- (A) 989 N
- (B) 1140 N
- (C) 1250 N
- (D) 1510 N

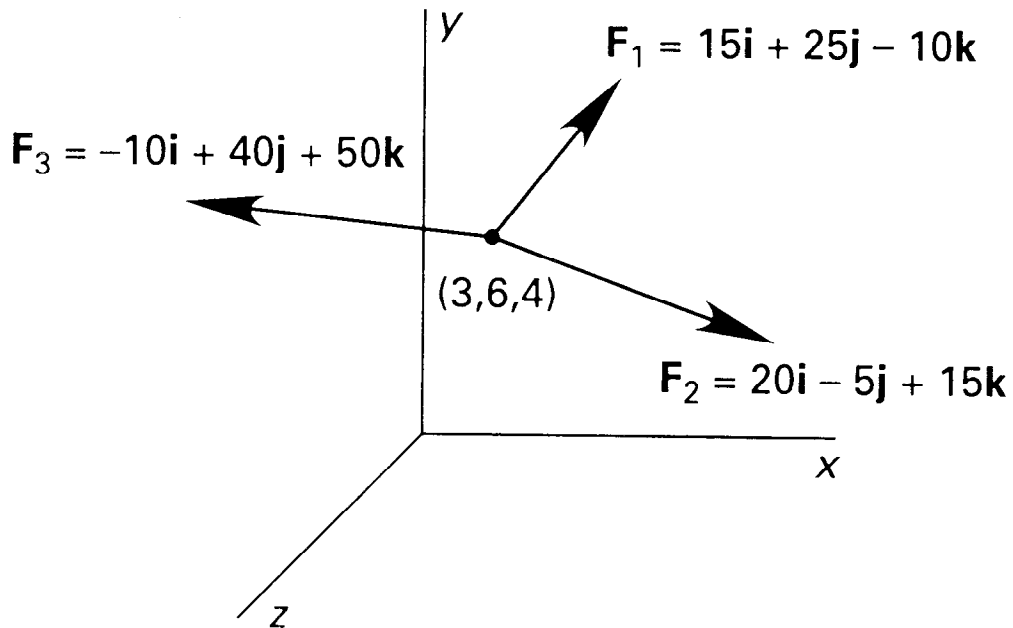
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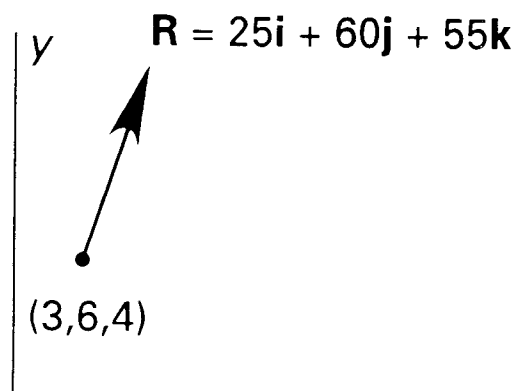
FE-STYLE EXAM PROBLEMS

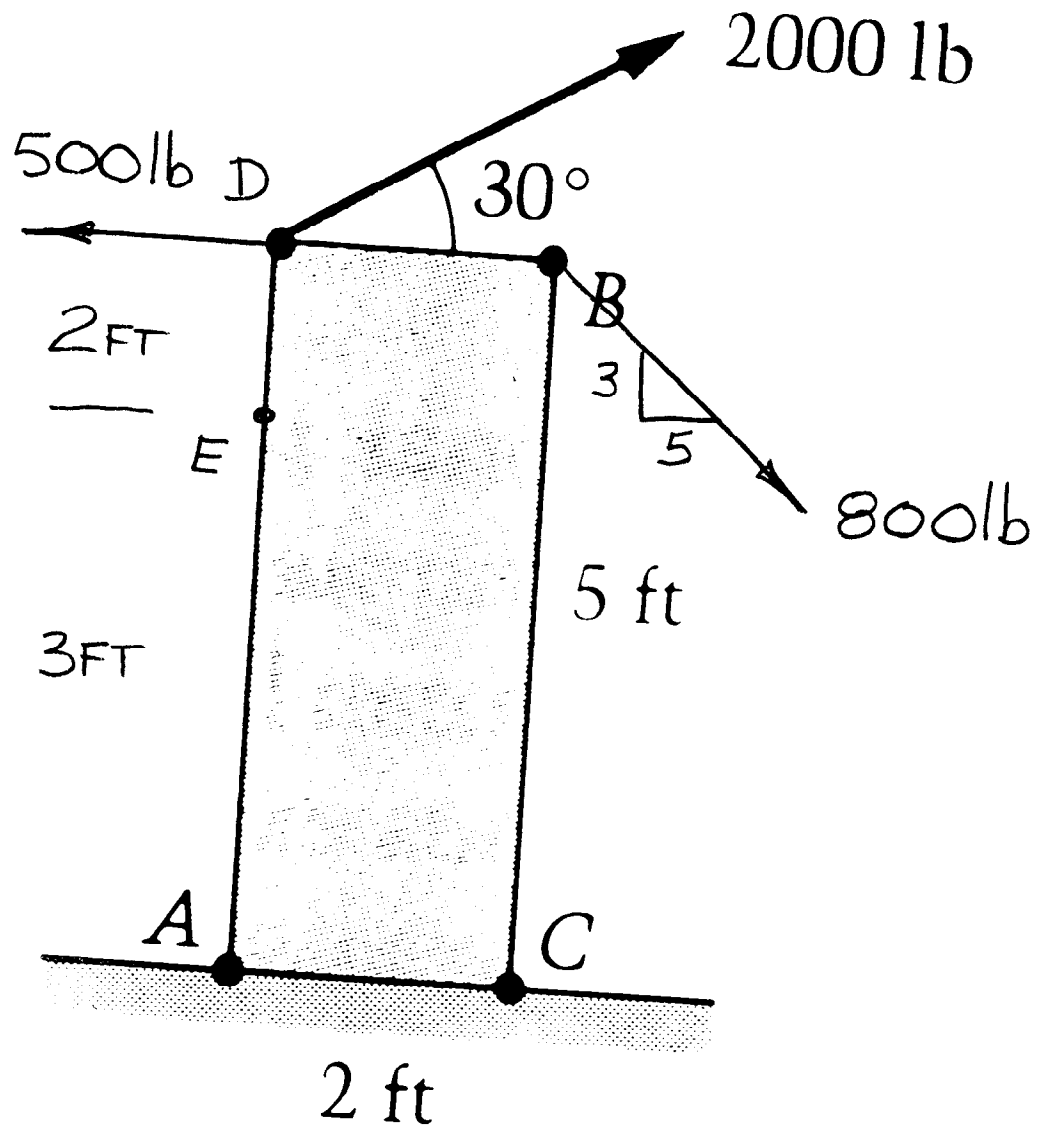
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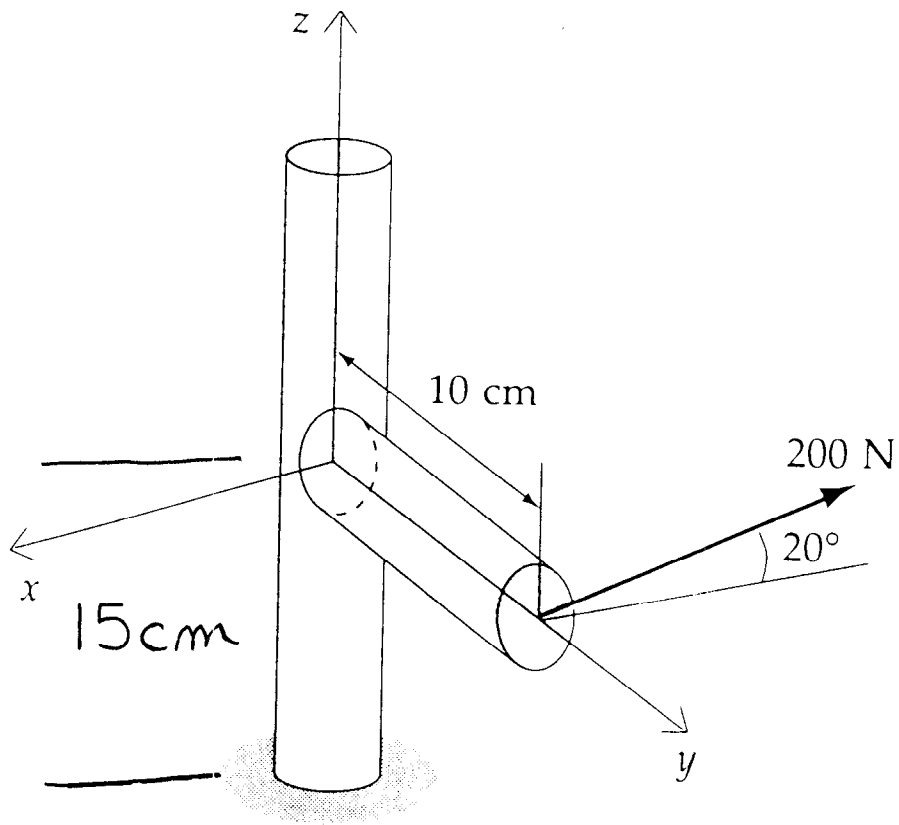
1. What is the resultant R of the system of forces shown?

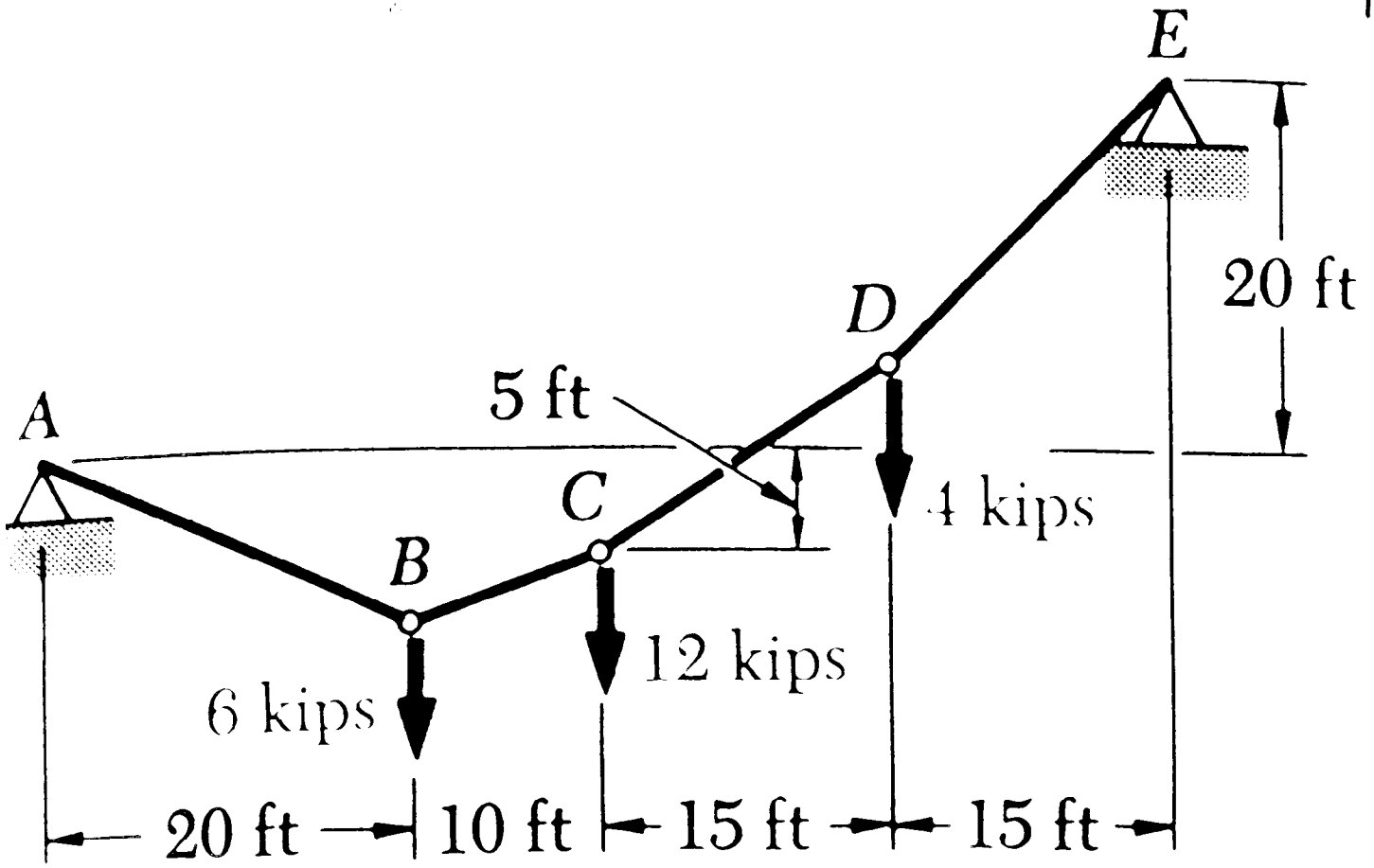


(A)

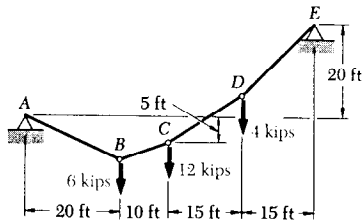








SAMPLE PROBLEM 7.8



The cable AE supports three vertical loads from the points indicated. If point C is 5 ft below the left support, determine (a) the elevations of points B and D , (b) the maximum slope and the maximum tension in the cable.

Solution. The reaction components A_x and A_y are determined as follows:

Free Body: Entire Cable

$$\begin{aligned}
 +\uparrow \Sigma M_E = 0: \\
 A_x(20 \text{ ft}) - A_y(60 \text{ ft}) + (6 \text{ kips})(40 \text{ ft}) + (12 \text{ kips})(30 \text{ ft}) + (4 \text{ kips})(15 \text{ ft}) = 0 \\
 20A_x - 60A_y + 660 = 0
 \end{aligned}$$

Free Body: ABC

$$\begin{aligned}
 +\uparrow \Sigma M_C = 0: \quad -A_x(5 \text{ ft}) - A_y(30 \text{ ft}) + (6 \text{ kips})(10 \text{ ft}) = 0 \\
 -5A_x - 30A_y + 60 = 0
 \end{aligned}$$

Solving the two equations simultaneously, we obtain

$$\begin{aligned}
 A_x = -18 \text{ kips} \quad A_x = 18 \text{ kips} \leftarrow \\
 A_y = +5 \text{ kips} \quad A_y = 5 \text{ kips} \uparrow
 \end{aligned}$$

a. *Elevation of Point B.* Considering the portion of cable AB as a free body, we write

$$\begin{aligned}
 +\uparrow \Sigma M_B = 0: \quad (18 \text{ kips})y_B - (5 \text{ kips})(20 \text{ ft}) = 0 \\
 y_B = 5.56 \text{ ft below A} \quad \blacktriangleleft
 \end{aligned}$$

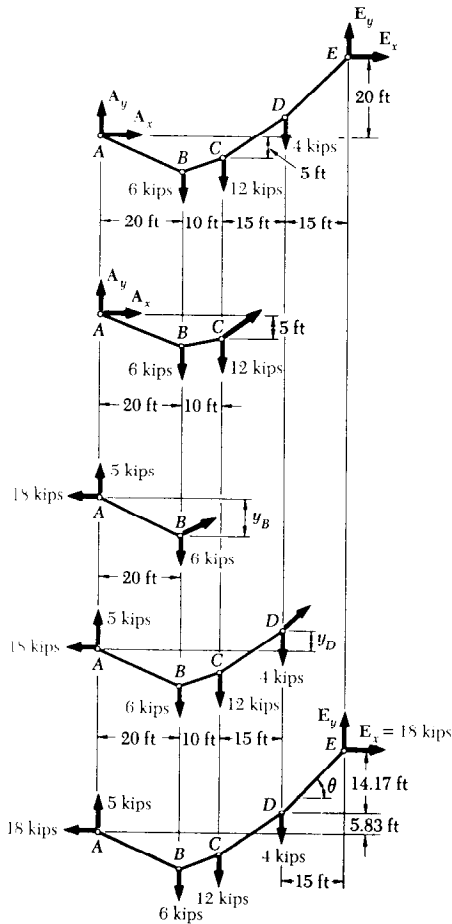
Elevation of Point D. Using the portion of cable $ABCD$ as a free body, we write

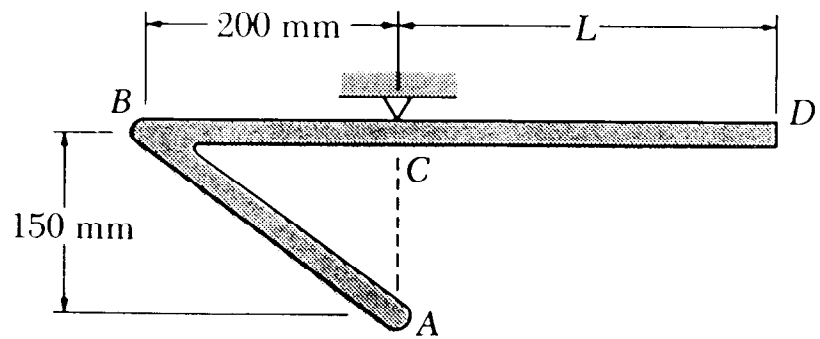
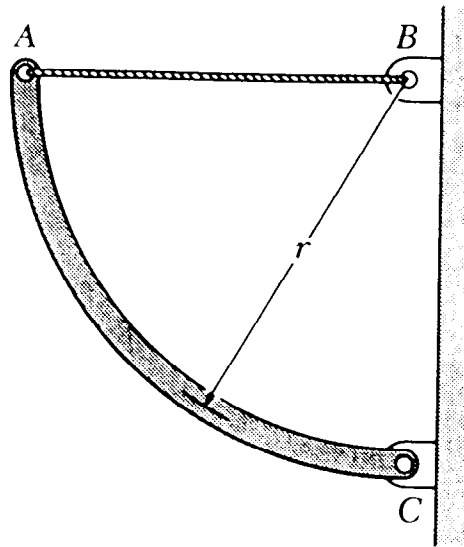
$$\begin{aligned}
 +\uparrow \Sigma M_D = 0: \\
 -(18 \text{ kips})y_D - (5 \text{ kips})(45 \text{ ft}) + (6 \text{ kips})(25 \text{ ft}) + (12 \text{ kips})(15 \text{ ft}) = 0 \\
 y_D = 5.83 \text{ ft above A} \quad \blacktriangleleft
 \end{aligned}$$

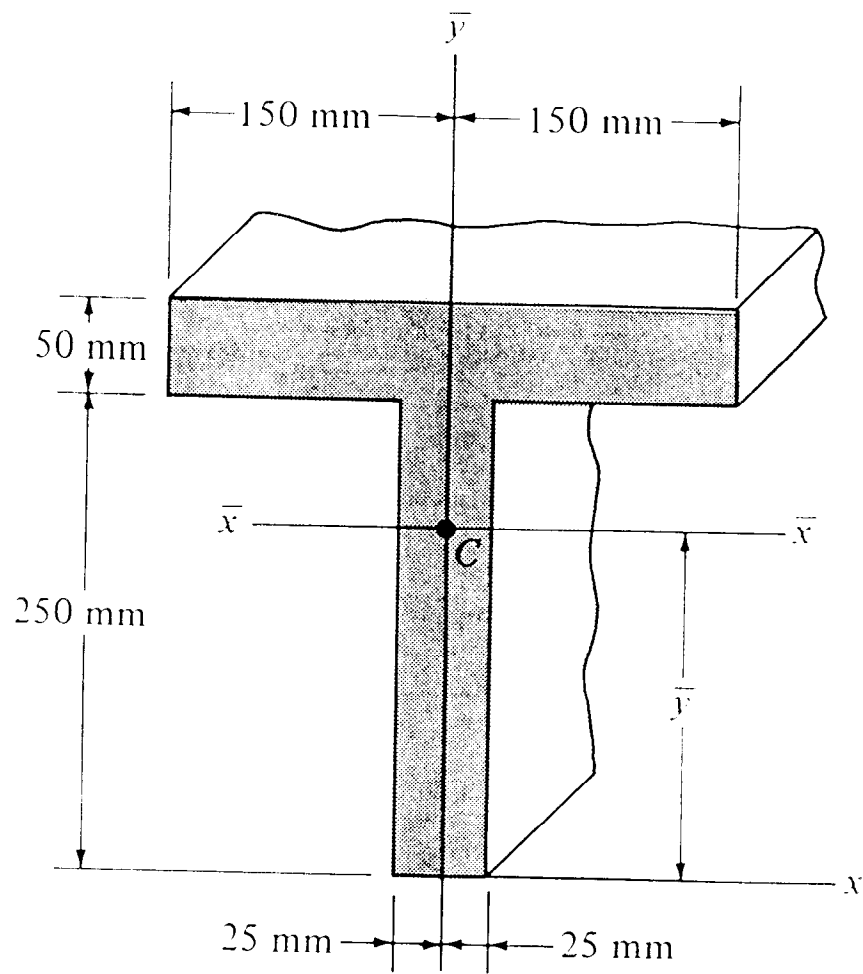
b. *Maximum Slope and Maximum Tension.* We observe that the maximum slope occurs in portion DE . Since the horizontal component of the tension is constant and equal to 18 kips, we write

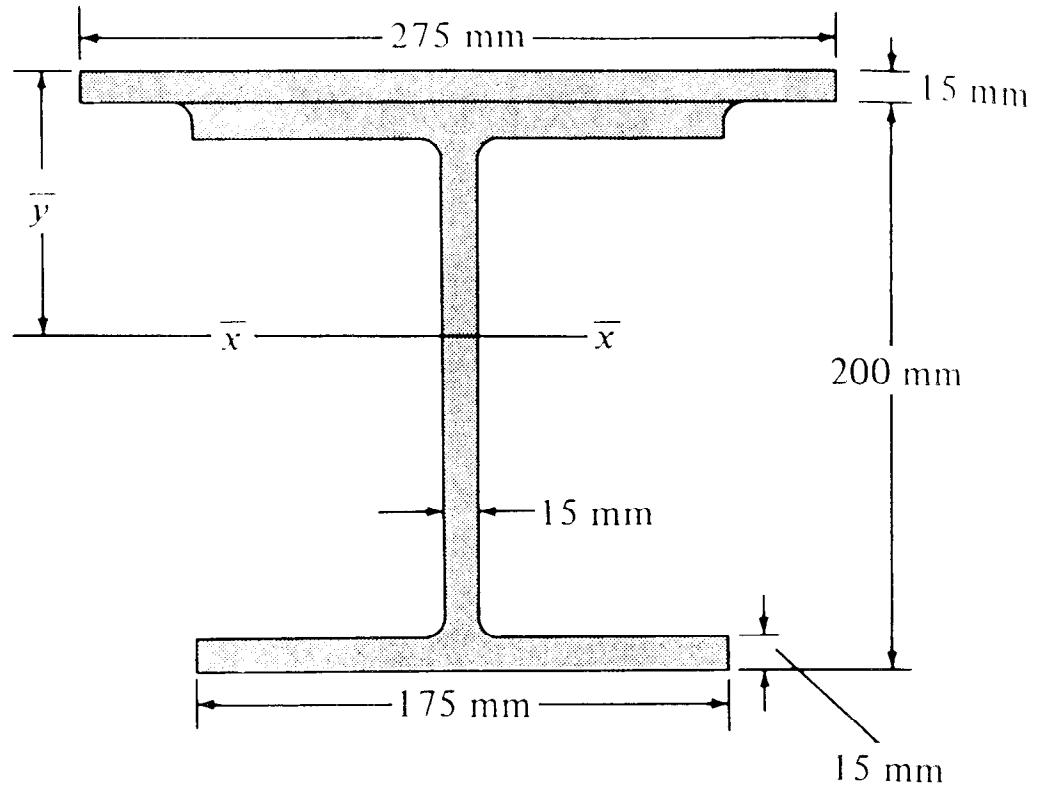
$$\tan \theta = \frac{14.17}{15 \text{ ft}} \quad \theta = 43.4^\circ \quad \blacktriangleleft$$

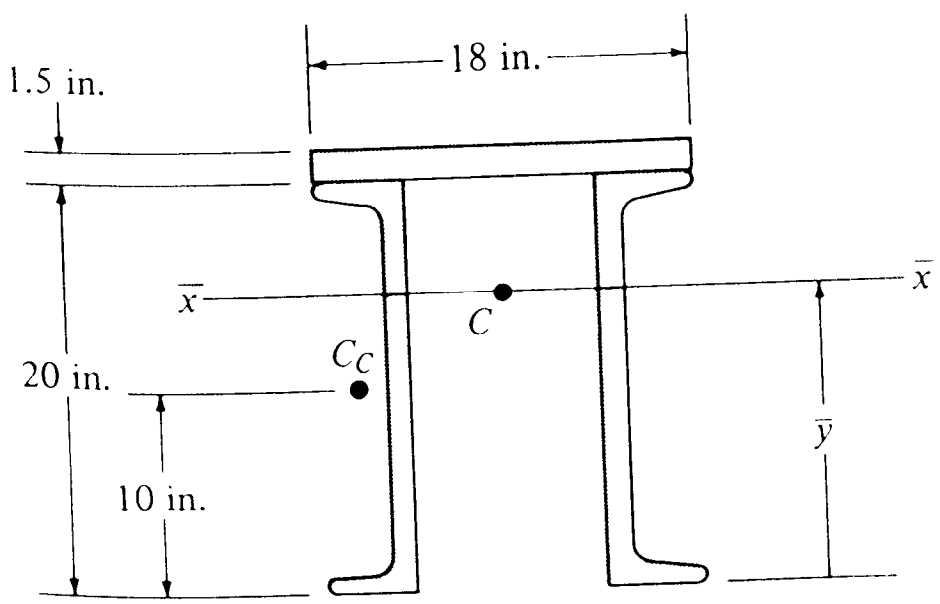
$$T_{\max} = \frac{18 \text{ kips}}{\cos \theta} \quad T_{\max} = 24.8 \text{ kips} \quad \blacktriangleleft$$

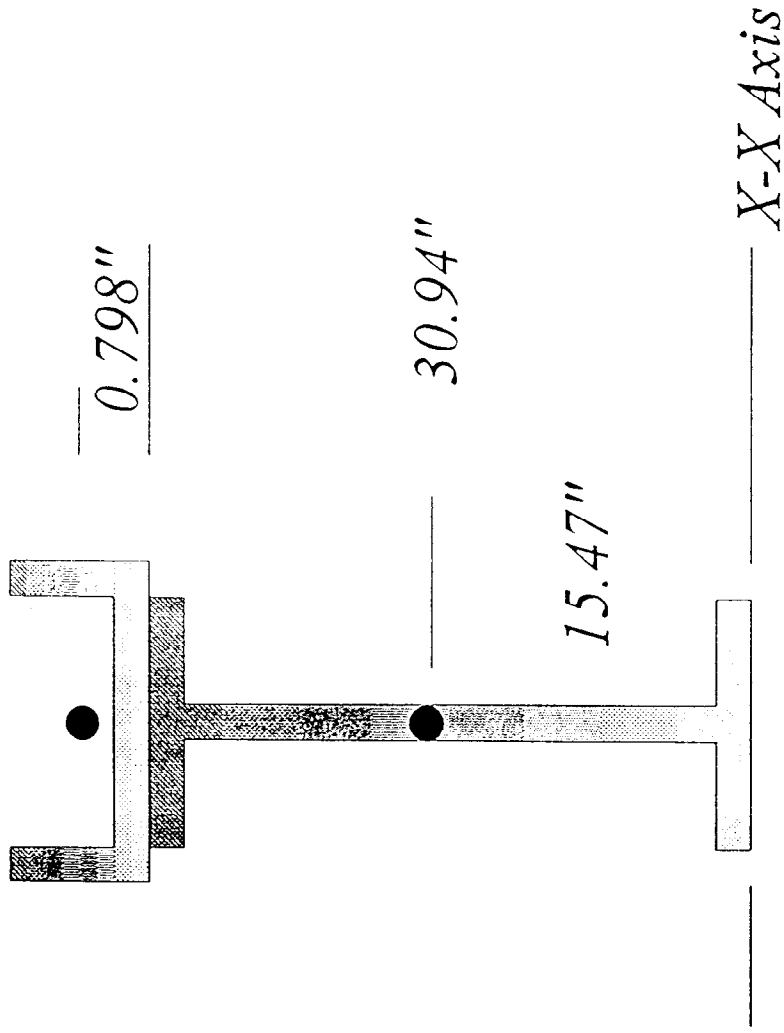




**Prob. 10-31**







Wide flange is W30x211:

$$\text{Area} = 62 \text{ in}^2, I_{xx} = 10300 \text{ in}^4$$

Channel is C15x50:

$$\text{Area} = 14.7 \text{ in}^2, I_{yy} = 11 \text{ in}^4$$

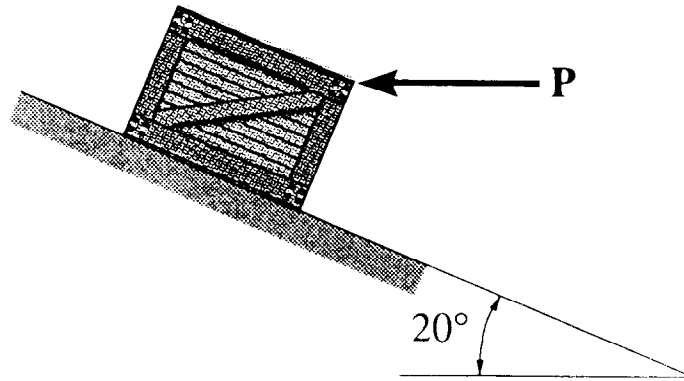
Product of Inertia. The integral

$$I_{xy} = \int xy \, dA$$

obtained by multiplying each element dA of an area A by its coordinates x and y and integrating over the area (Fig. 9.14), is known as the *product of inertia* of the area A with respect to the x and y axes. Unlike the moments of inertia I_x and I_y , the product of inertia I_{xy} may be either positive or negative.

When one or both of the x and y axes are axes of symmetry for the area A , the product of inertia I_{xy} is zero. Consider, for example, the channel section shown in Fig. 9.15. Since this section is symmetrical with respect to the x axis, we can associate to each element dA of coordinates x and y an element dA' of coordinates x and $-y$. Clearly, the contributions of any pair of elements chosen in this way cancel out, and the integral (9.12) reduces to zero.

4-73. Determine the range of values for which the horizontal force P will prevent the crate from slipping down or up the inclined plane. Take $\mu_s = 0.1$.



4-74. The refrigerator has a weight of 200 lb and a center of gravity at G . Determine the force P required to move it. Will the refrigerator tip or slip? Take $\mu_s = 0.4$.

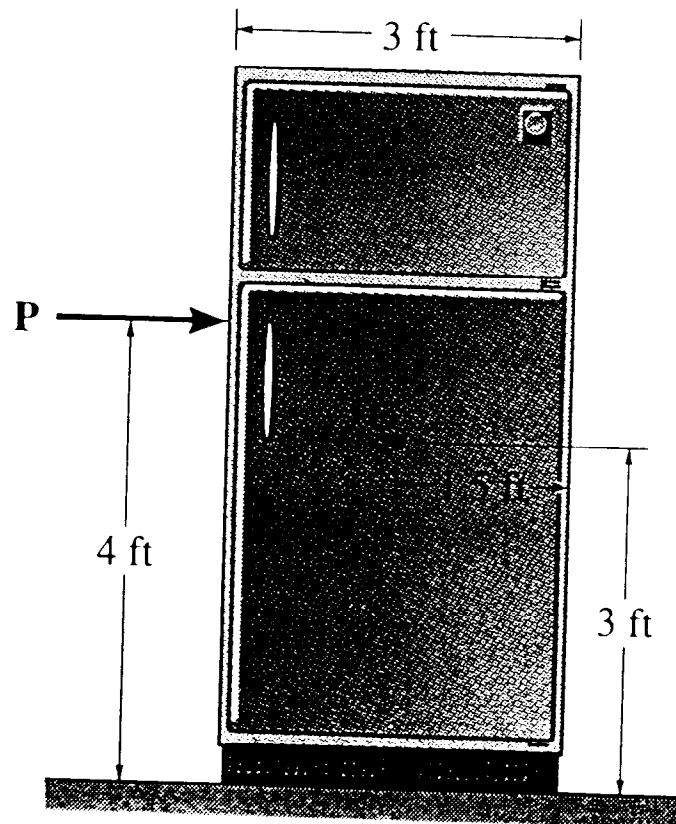
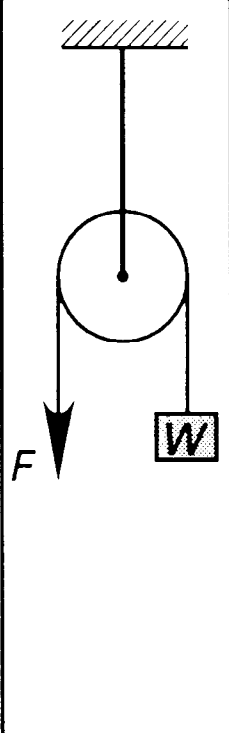
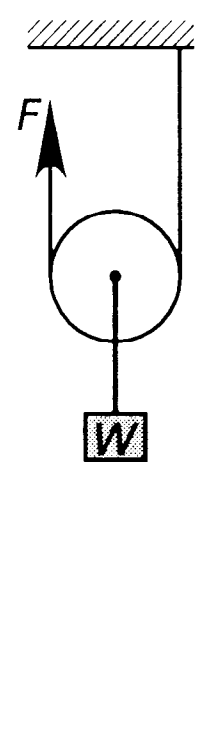
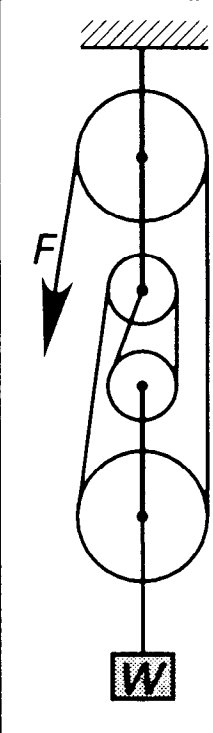
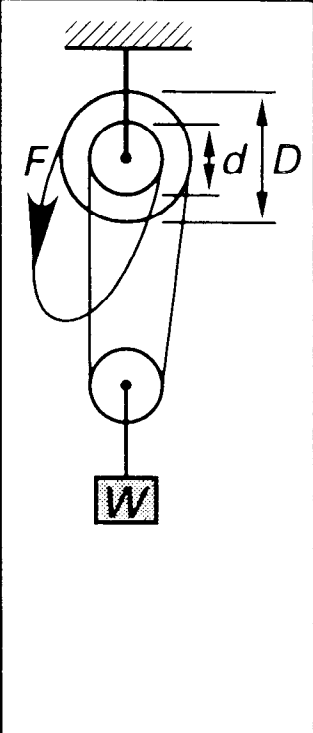


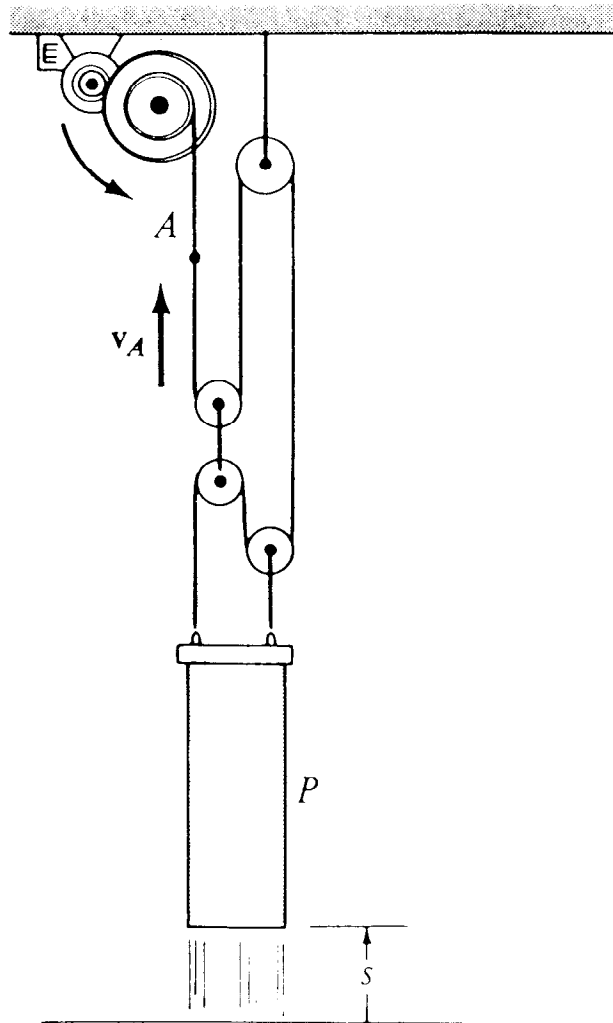
Figure 12.1 Mechanical Advantage of Rope-Operated Machines

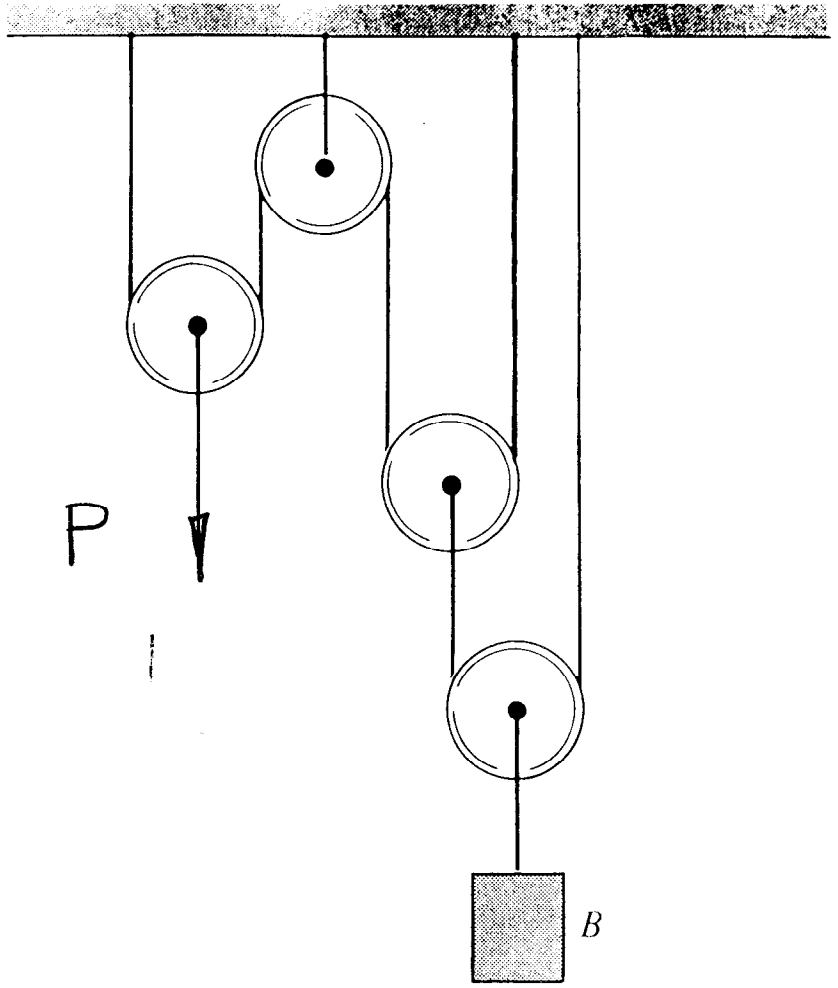
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12-2

	fixed sheave	free sheave	ordinary pulley block (n sheaves)	differential pulley block
				
F_{ideal}	W	$\frac{W}{2}$	$\frac{W}{n}$	$\frac{W}{2} \left(1 - \frac{d}{D} \right)$

If the pulley is attached by a bracket to a fixed location, it is said to be a *fixed pulley*. If the pulley is attached to a load, or if the pulley is free to move, it is known as a *free pulley*.

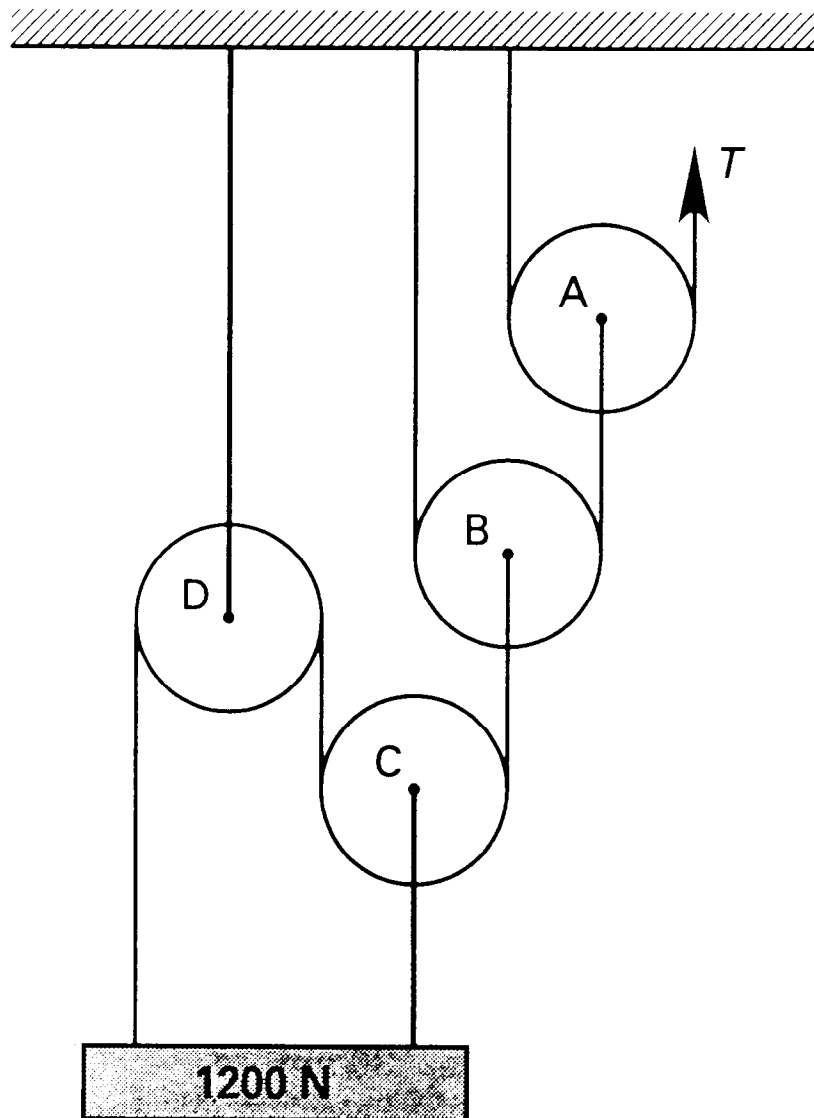
Most simple problems disregard friction and assume that all ropes (fiber ropes, wire ropes, chains, belts, etc.) are parallel. In such cases, the pulley advantage is





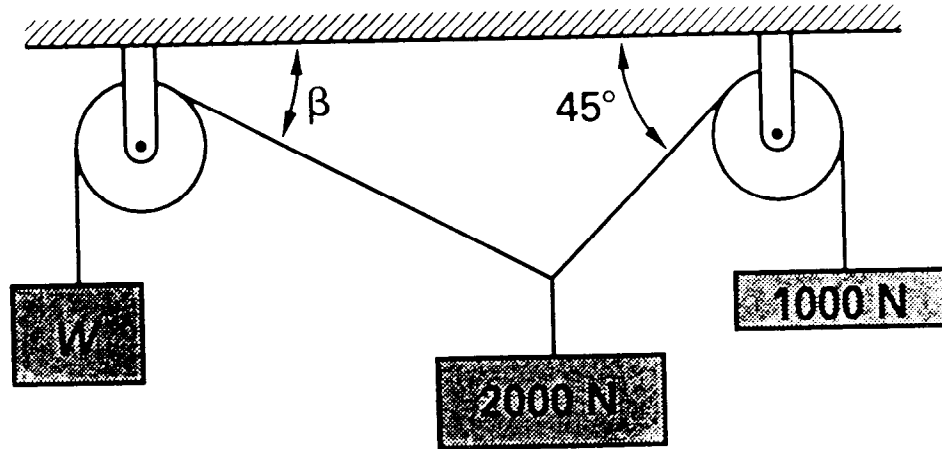
SAMPLE PROBLEMS

1. Find the tension, T , that must be applied to pulley A to lift the 1200 N weight.



- (A) 100 N
- (B) 300 N
- (C) 400 N
- (D) 600 N

1. The system shown is in static equilibrium. Find W .

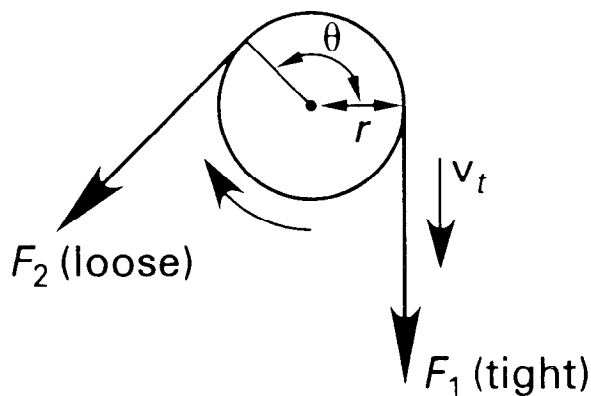


- (A) 830 N
(B) 1000 N
(C) 1500 N
(D) 1700 N

Friction between a belt, rope, or band wrapped around a pulley or sheave is responsible for the transfer of torque. Except when stationary, one side of the belt (the tight side) will have a higher tension than the other (the slack side). The basic relationship between the belt tensions and the coefficient of friction neglects centrifugal effects and is given by Eq. 12.4. F_1 is the tension on the tight side (direction of movement); F_2 is the tension on the other side. The *angle of wrap*, θ , must be expressed in radians.

$$F_1 = F_2 e^{\mu\theta} \tag{12.4}$$

Figure 12.3 Belt Friction

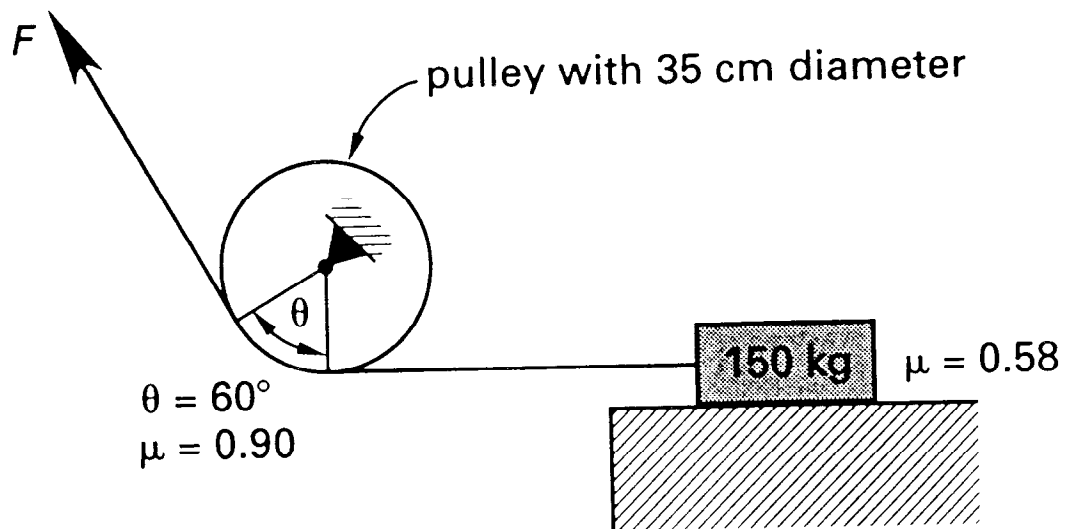


The net transmitted torque is

$$T = (F_1 - F_2)r \tag{12.5}$$

The power transmitted to a belt running at tangential velocity v_t is

Problems 2 and 3 refer to the following illustration.

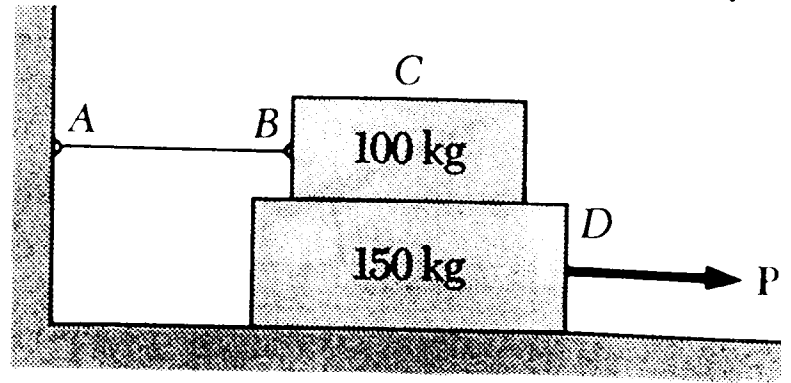


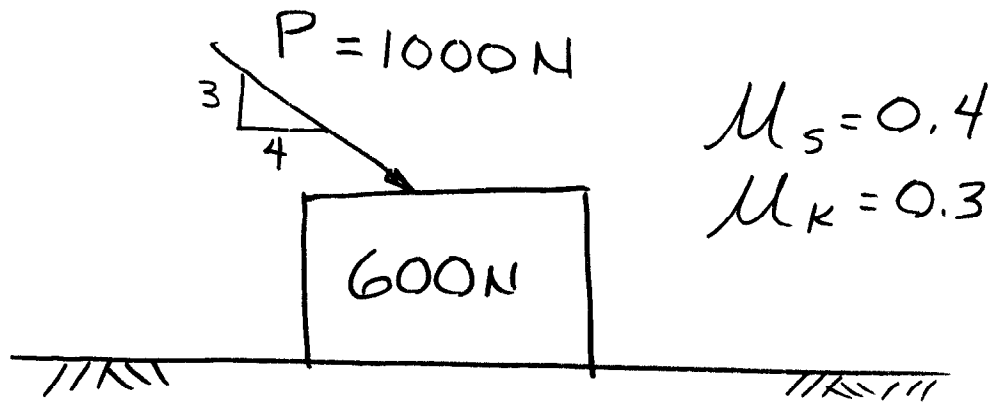
$$\mu_s = 0.3$$

$$\mu_k = 0.25$$

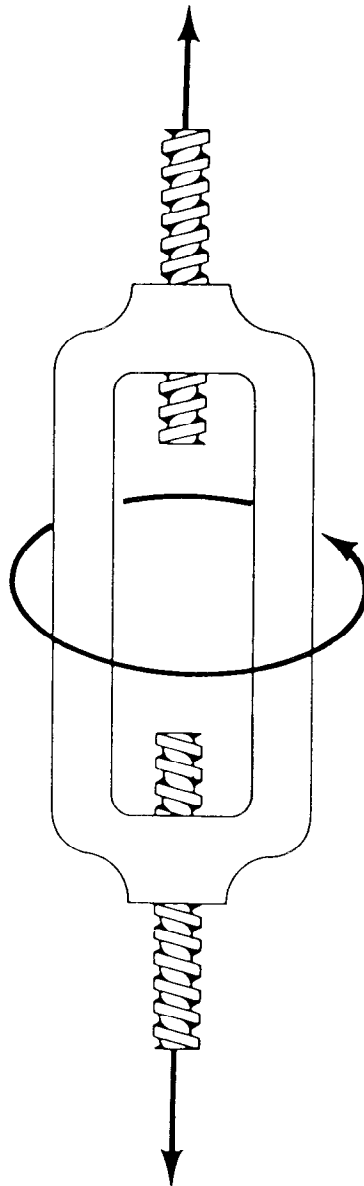
$$P = 900 \text{ N}$$

WILL BLOCK MOVE?





WILL BLOCK MOVE?



Example 9-8

The turnbuckle shown in Fig. 9-20 has a square thread with a mean radius of 5 mm and a pitch of 2 mm. If the coefficient of friction between the screw and the turnbuckle is $\mu_s = 0.25$, determine the moment M that must be applied to draw the end screws closer together. Is the turnbuckle self-locking?

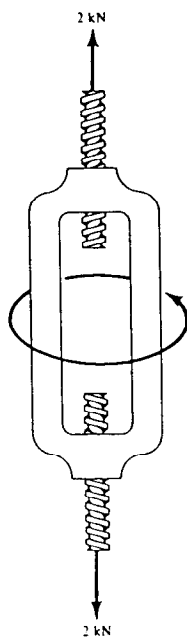


Fig. 9-20

Solution

The moment may be obtained by using Eq. 9-3. Since friction at two screws must be overcome, this requires

$$M = 2[Wr \tan(\phi_s + \theta_p)] \quad (1)$$

Here, $W = 2000 \text{ N}$, $r = 5 \text{ mm}$, $\phi_s = \tan^{-1} \mu_s = \tan^{-1}(0.25) = 14.04^\circ$, and $\theta_p = \tan^{-1}(p/2\pi r) = \tan^{-1}(2 \text{ mm}/[2\pi(5 \text{ mm})]) = 3.64^\circ$. Substituting these values into Eq. (1) and solving gives

$$M = 2\{(2000 \text{ N})(5 \text{ mm}) \tan(14.04^\circ + 3.64^\circ)\}$$

$$M = 6375.1 \text{ N} \cdot \text{mm} = 6.38 \text{ N} \cdot \text{m} \quad \text{Ans.}$$

When the moment is *removed*, the turnbuckle will be self-locking; i.e., it will not unscrew, since $\phi_s > \theta_p$.

9. A box has uniform density and a total weight of 600 N. It is suspended by three equal-length cables, AE, BE, and CE, as shown. Point E is 0.5 m directly above the center of the box's top surface. What is the tension in cable CE?

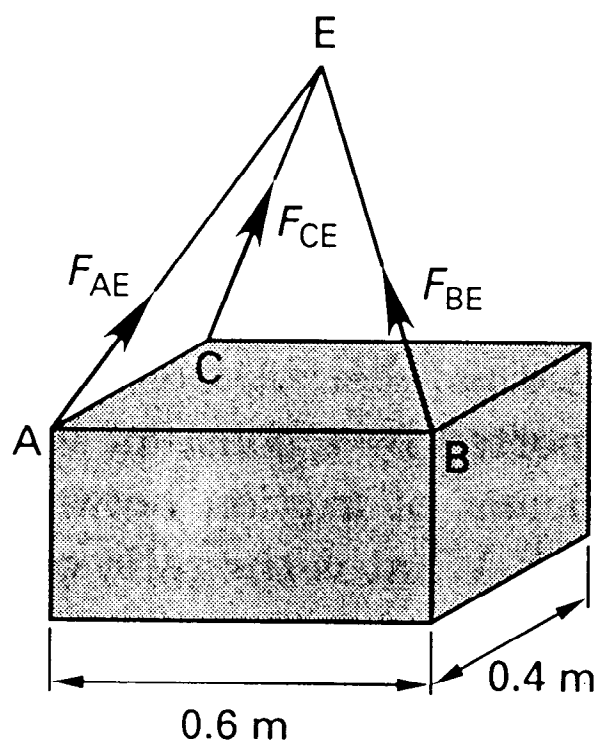
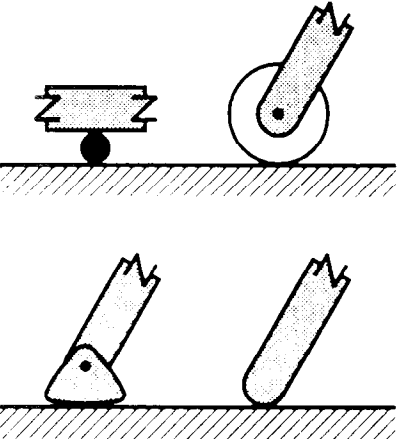
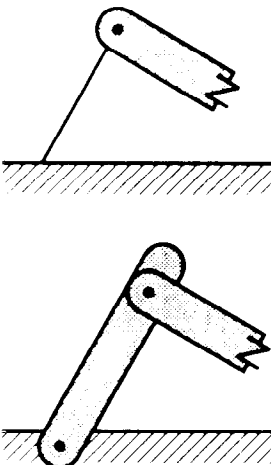

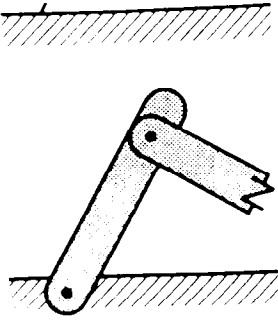
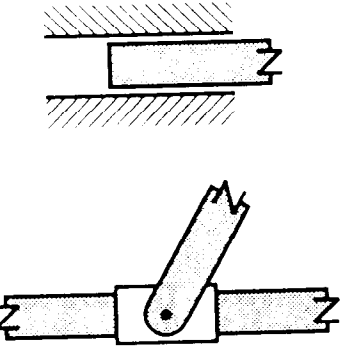
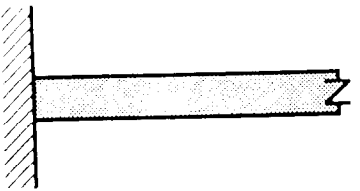
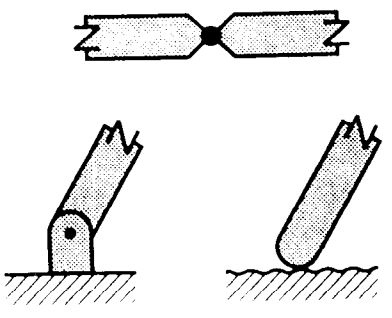


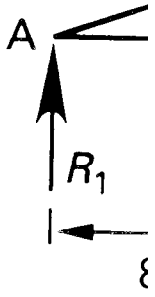
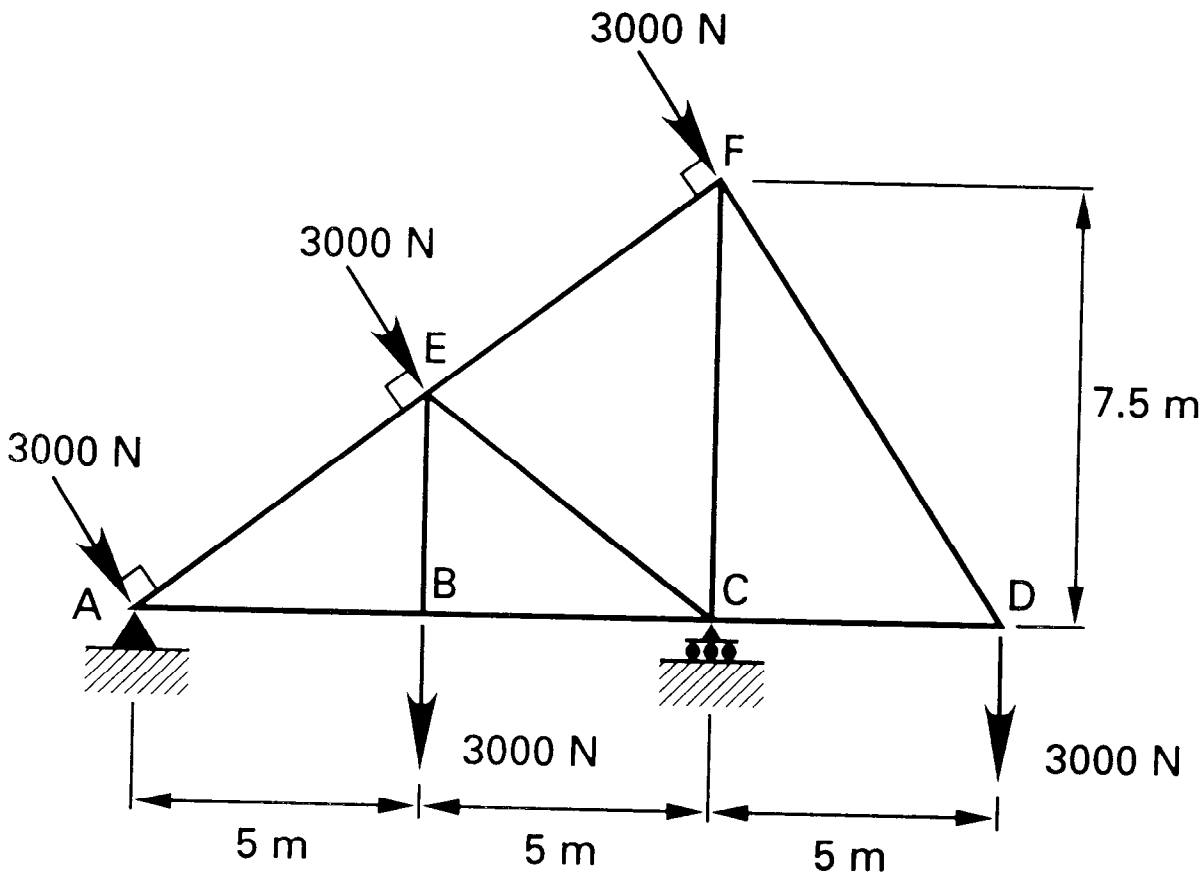
Table 10.1 Types of Two-Dimensional Supports

type of support	reactions and moments	number of unknowns ^(a)
<p>simple, roller, rocker, ball, or frictionless surface</p>  <p>The diagrams show: 1. A rectangular block on a horizontal surface with a single roller underneath. 2. A cylindrical roller on a horizontal surface. 3. A triangular wedge on a horizontal surface. 4. A cylindrical roller on a horizontal surface.</p>	<p>reaction normal to surface, no moment</p>	<p>1</p>
<p>cable in tension, or link</p>  <p>The diagrams show: 1. A cable attached to a horizontal surface and a vertical link. 2. A cable attached to a horizontal surface and a vertical link.</p>	<p>reaction in line with cable or link, no moment</p>	<p>1</p>
<p>frictionless guide or collar</p>  <p>The diagram shows a horizontal surface with a hatched area below it, representing a guide or collar.</p>		

	<p>with cable or link, no moment</p>	<p>40</p>
<p>frictionless guide or collar</p> 	<p>reaction normal to rail, no moment</p>	<p>1</p>
<p>built-in, fixed support</p> 	<p>two reaction components, one moment</p>	<p>3</p>
<p>frictionless hinge, pin connection, or rough surface</p> 	<p>reaction in any direction, no moment</p>	<p>2</p>

(a) The number of unknowns is valid for two-dimensional problems only.

3. Determine the force in member BC.



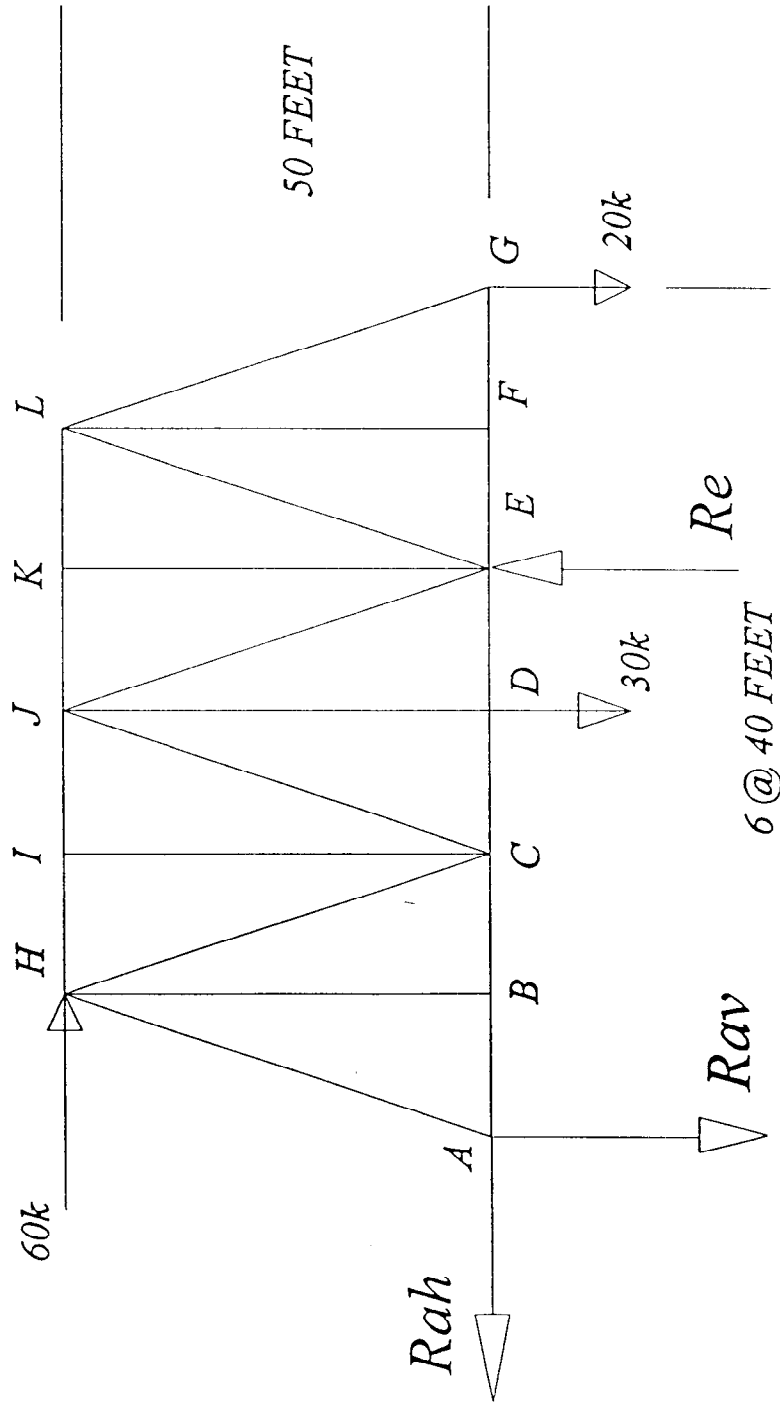
5. Wh

- (A)
- (B)
- (C)
- (D)

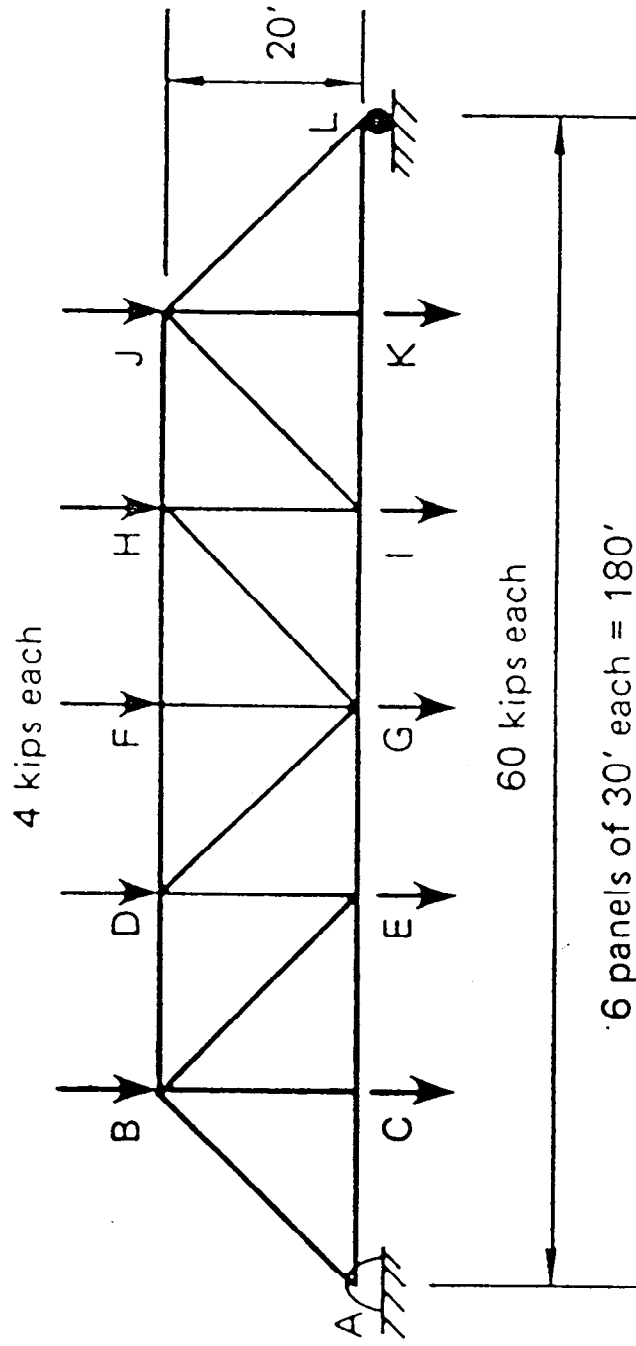
6. Wha

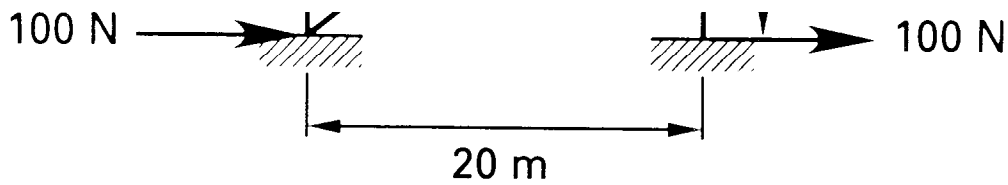
- (A)
- (B)
- (C)

- (A) 0
- (B) 1000 N (compression)
- (C) 1500 N (tension)
- (D) 2500 N (tension)



4. Find the forces in members DE and HJ.





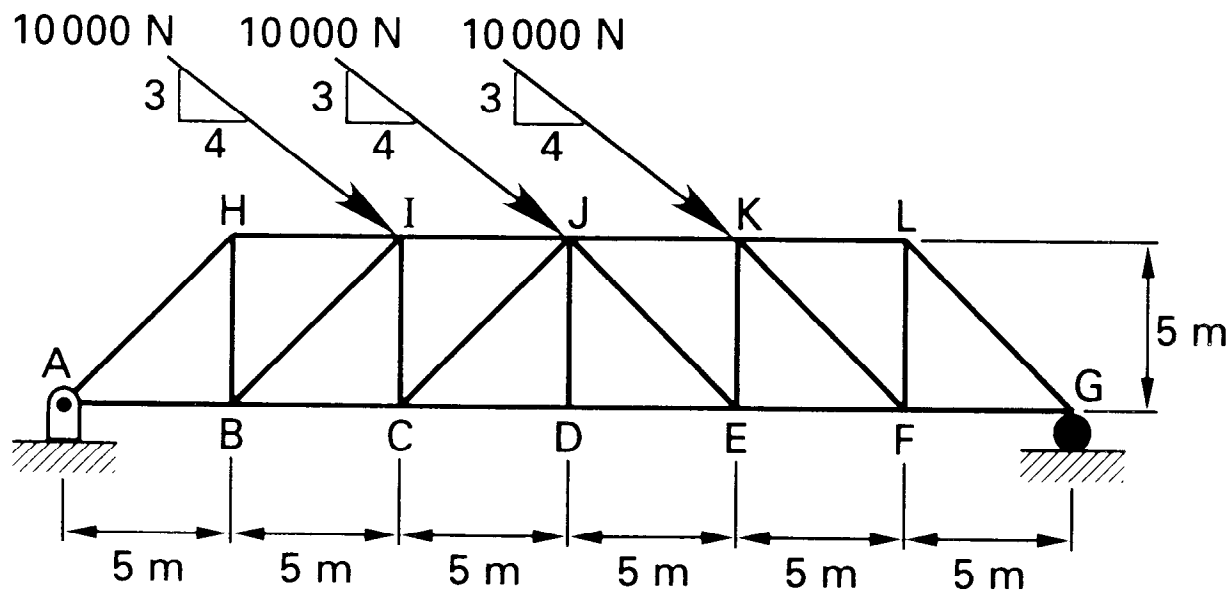
44

10

- (A) 0
- (B) 160 N
- (C) 200 N
- (D) 250 N

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8. The pedestrian bridge truss shown has 10 000 N applied loads at points I, J, and K. What is the force in member IJ?



- (A) 8000 N (compression)
- (B) 8000 N (tension)

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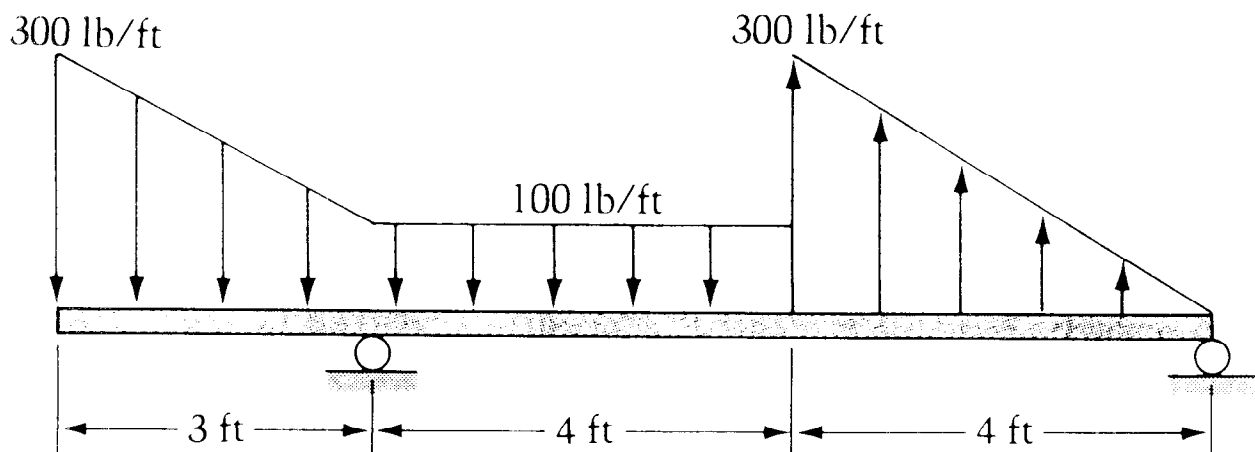


Figure P3.155

3.156

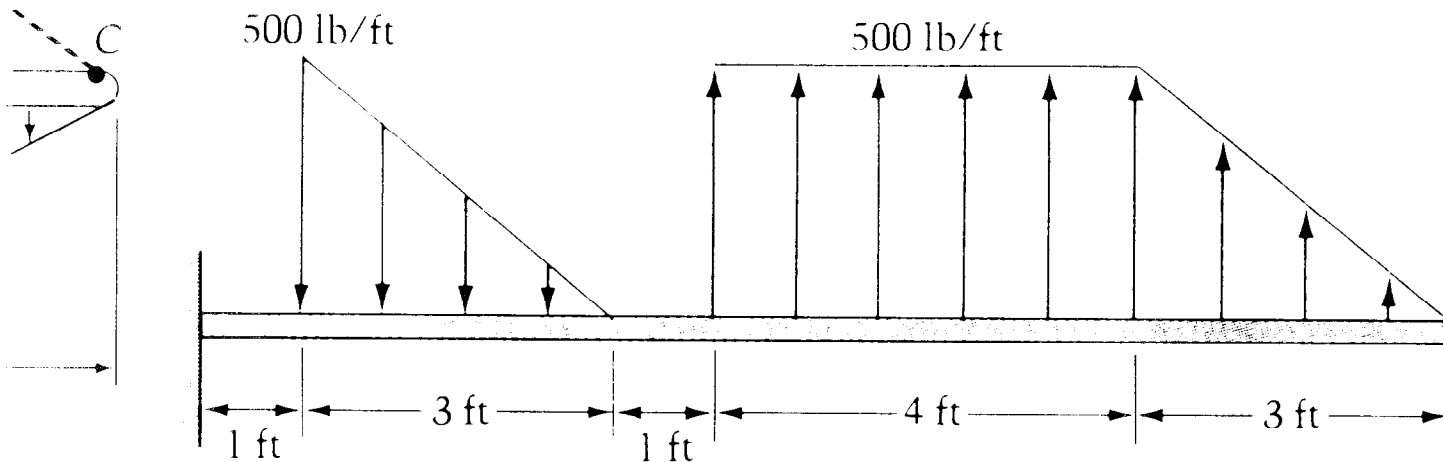
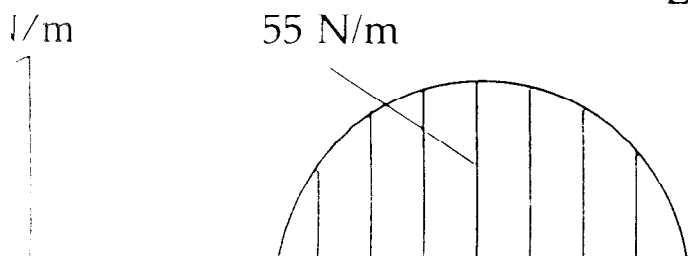
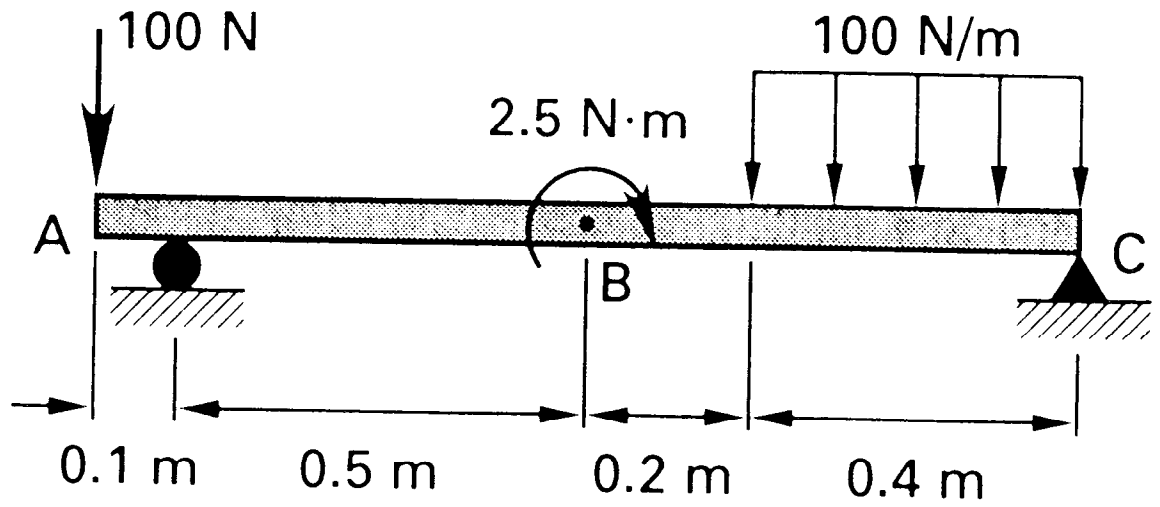


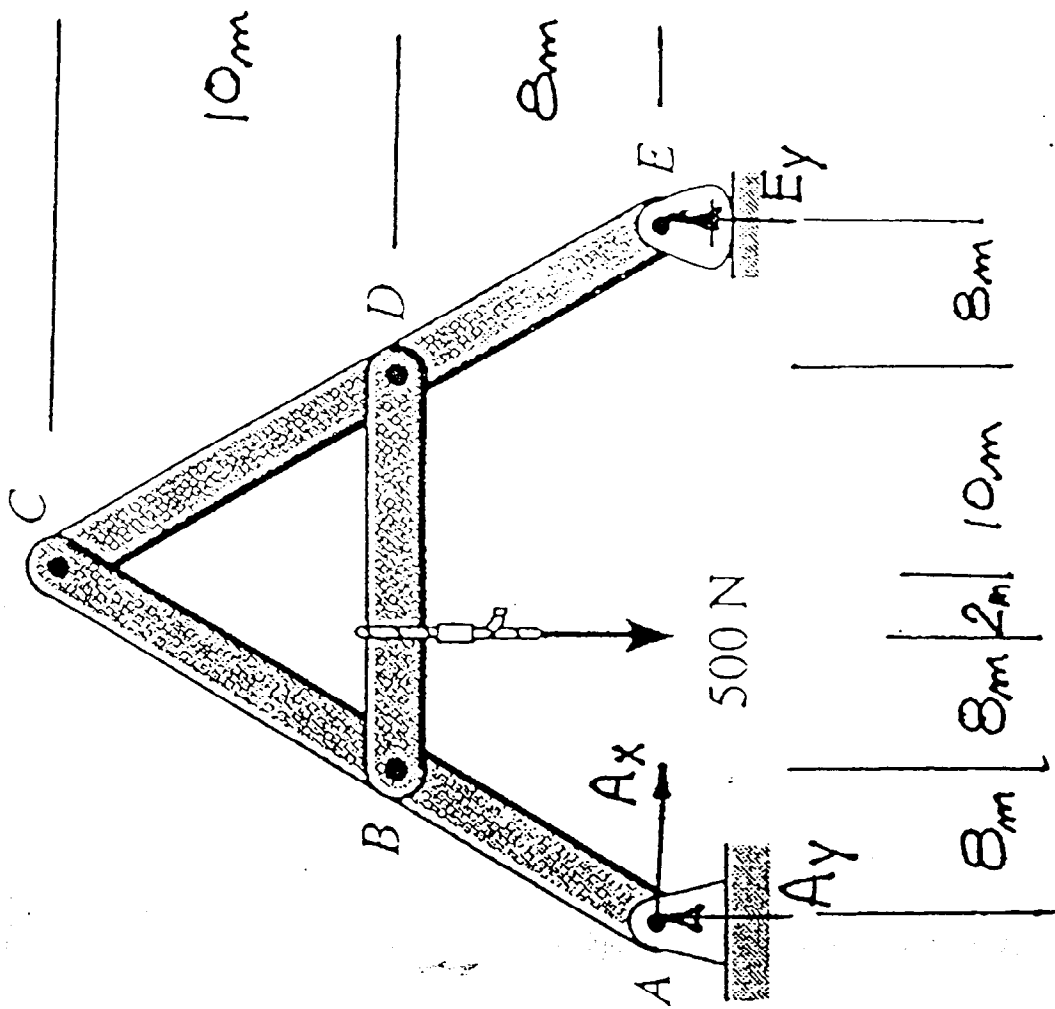
Figure P3.156

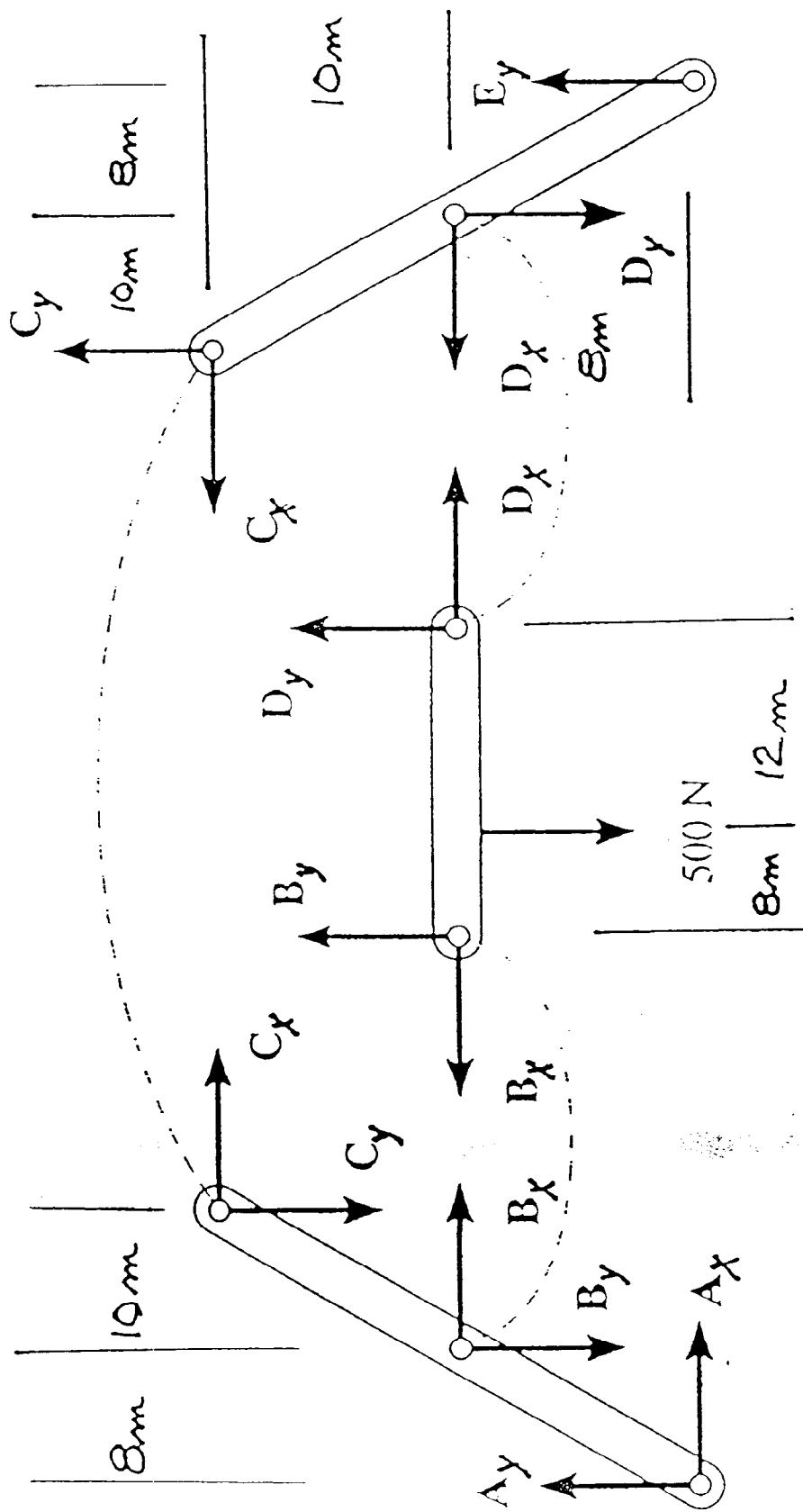
***3.157**

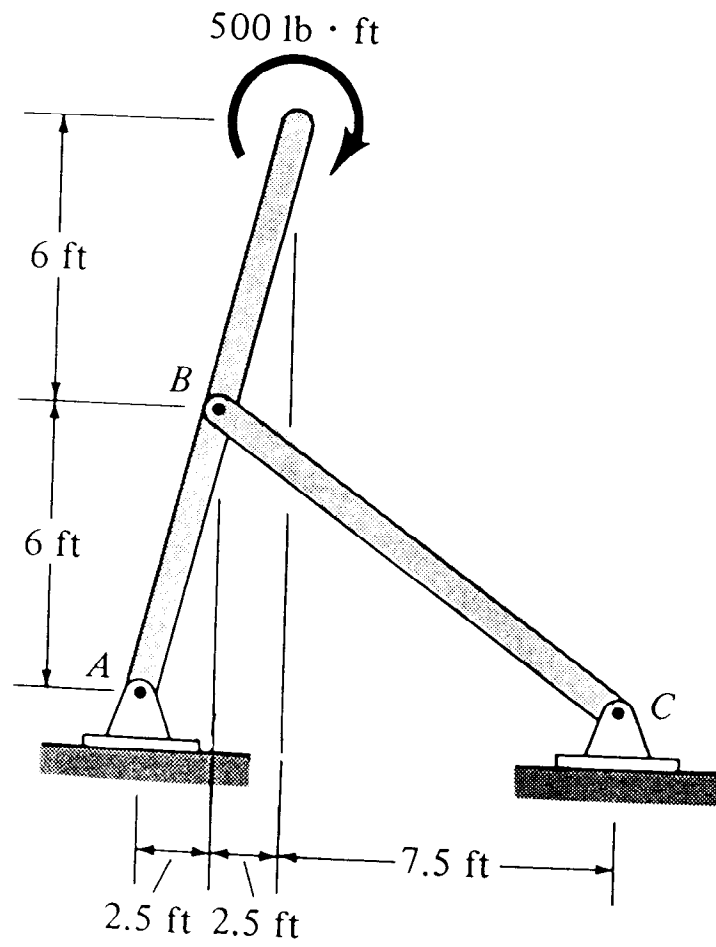


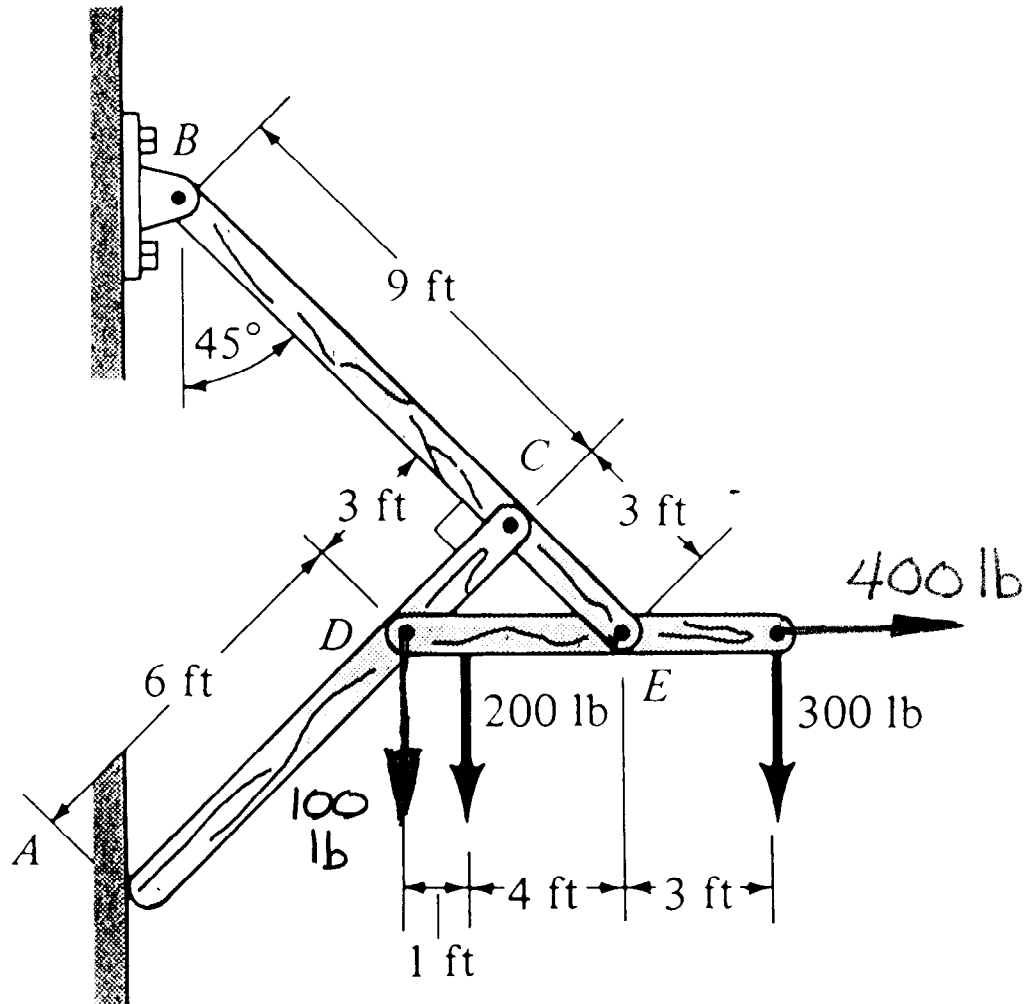
Loading curves are
semicircles, radii
1.1 m and 0.7 m

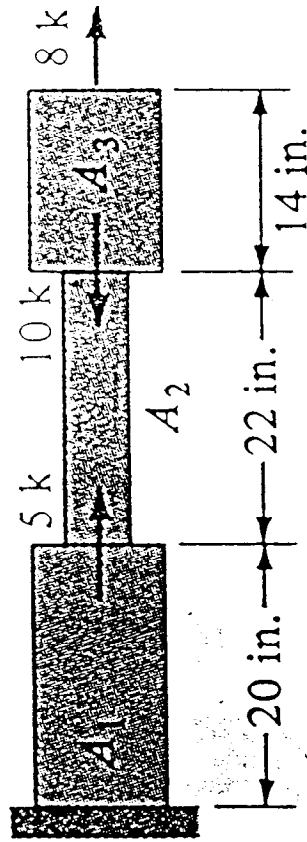


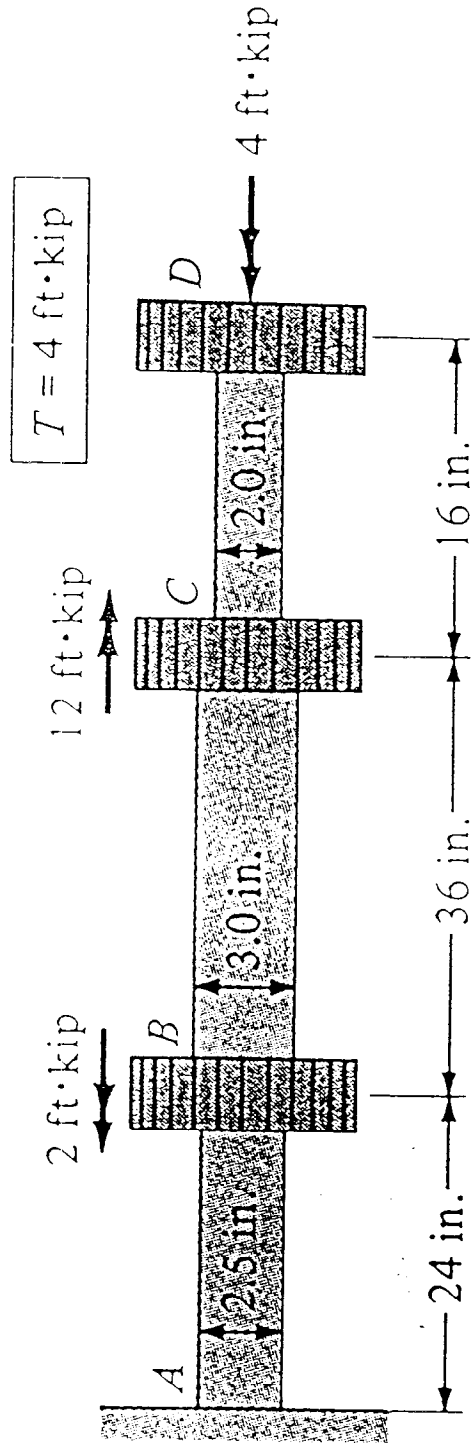


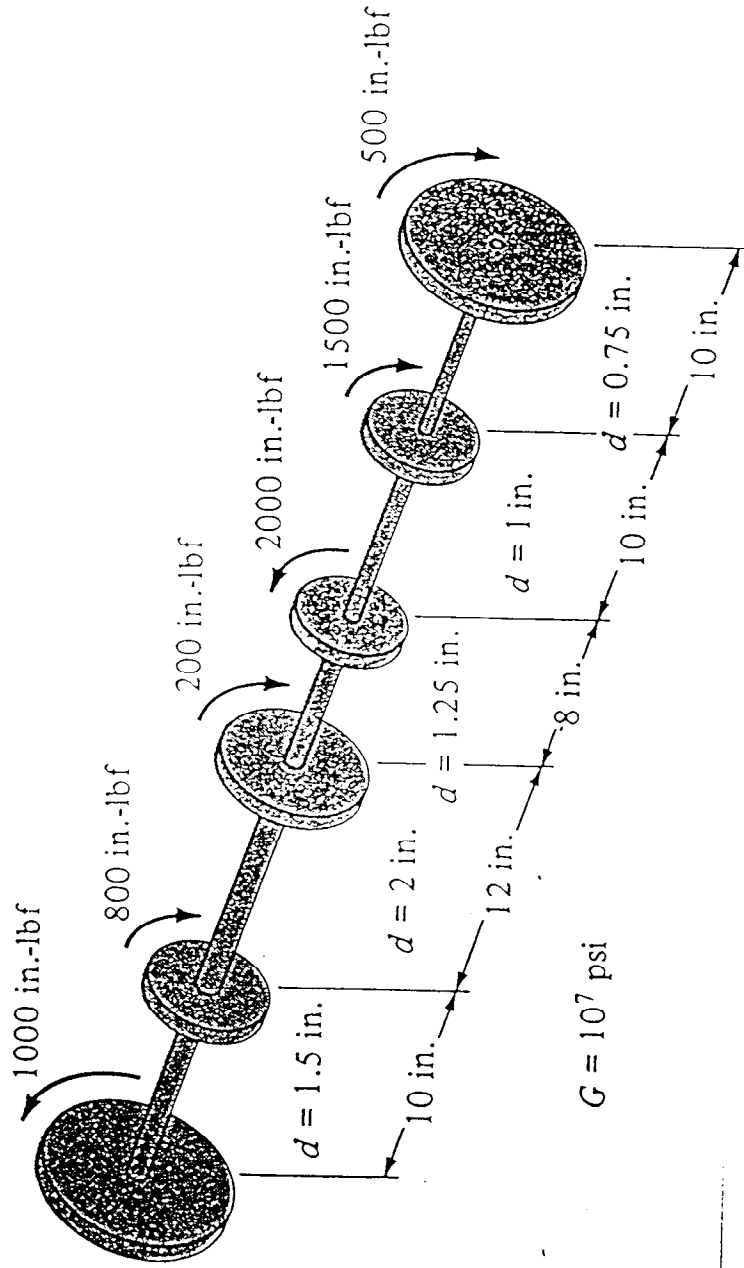


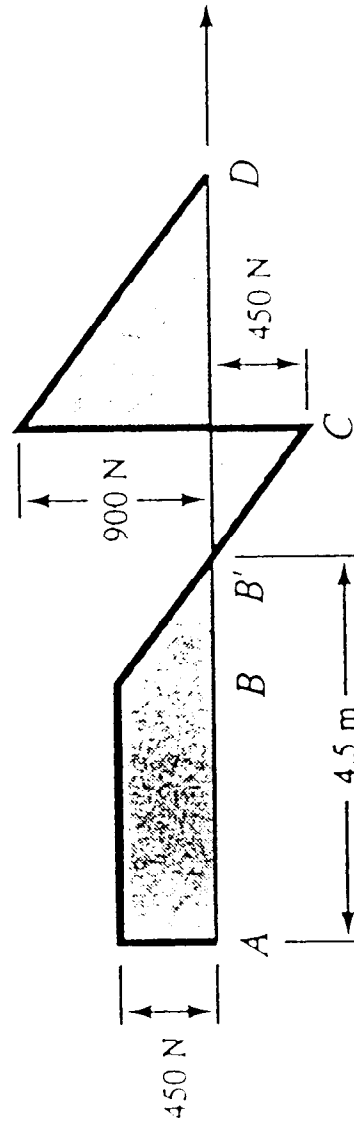
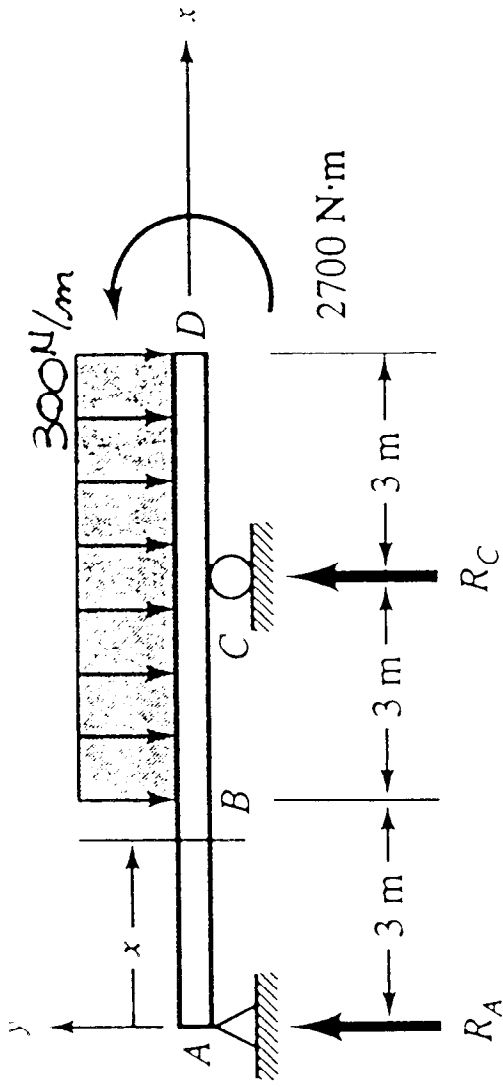




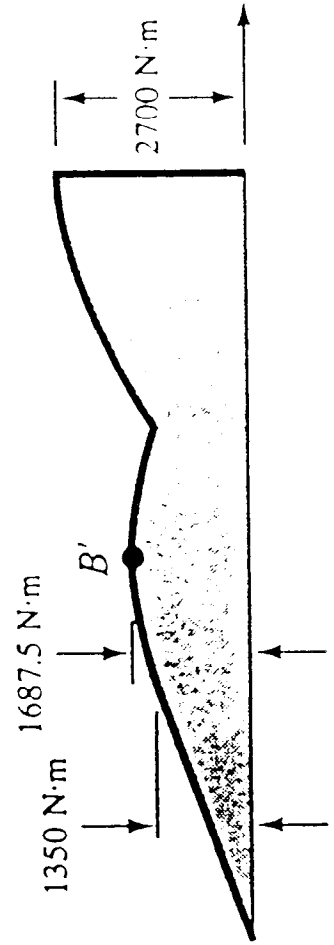




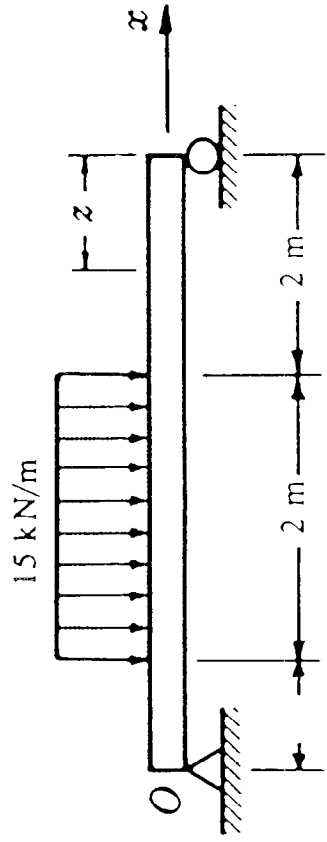




(a) Shear

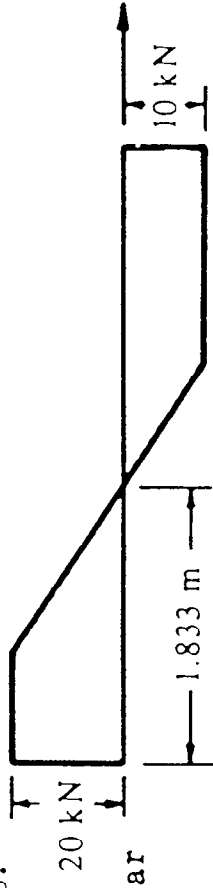


(b) Bending Moment

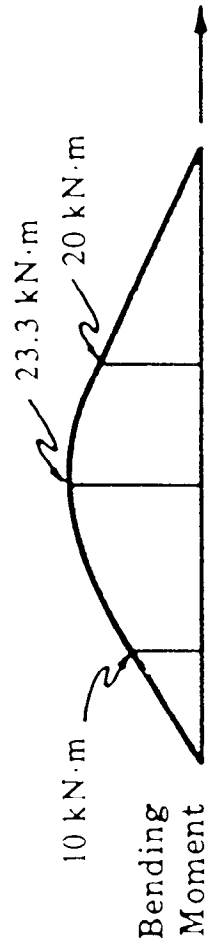


0.5 m

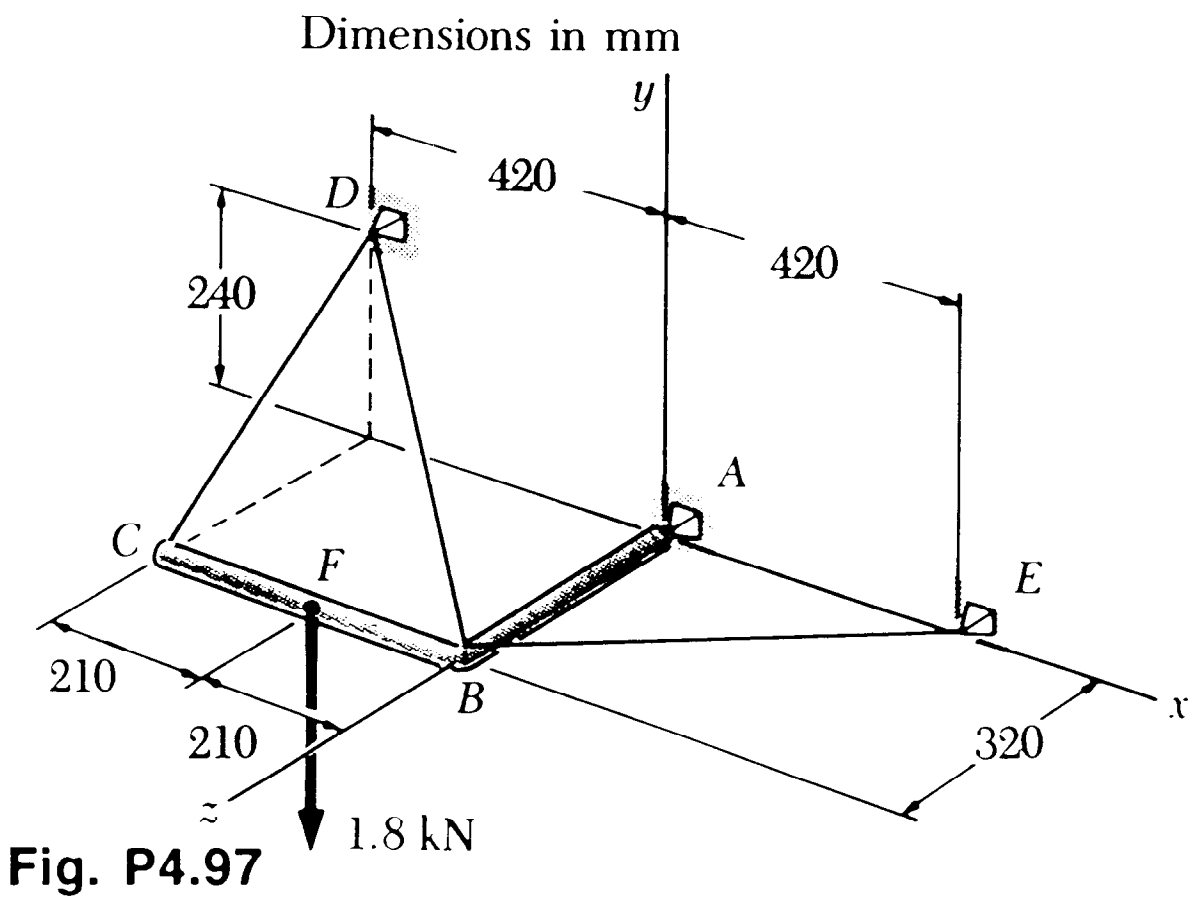
Ans.



Shear



Bending Moment



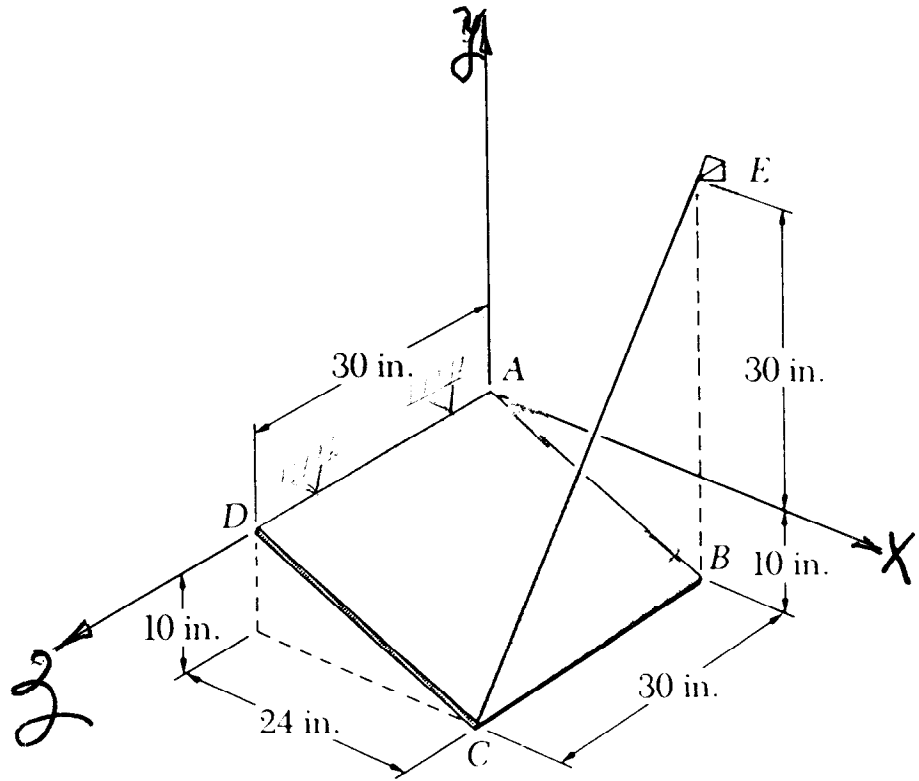
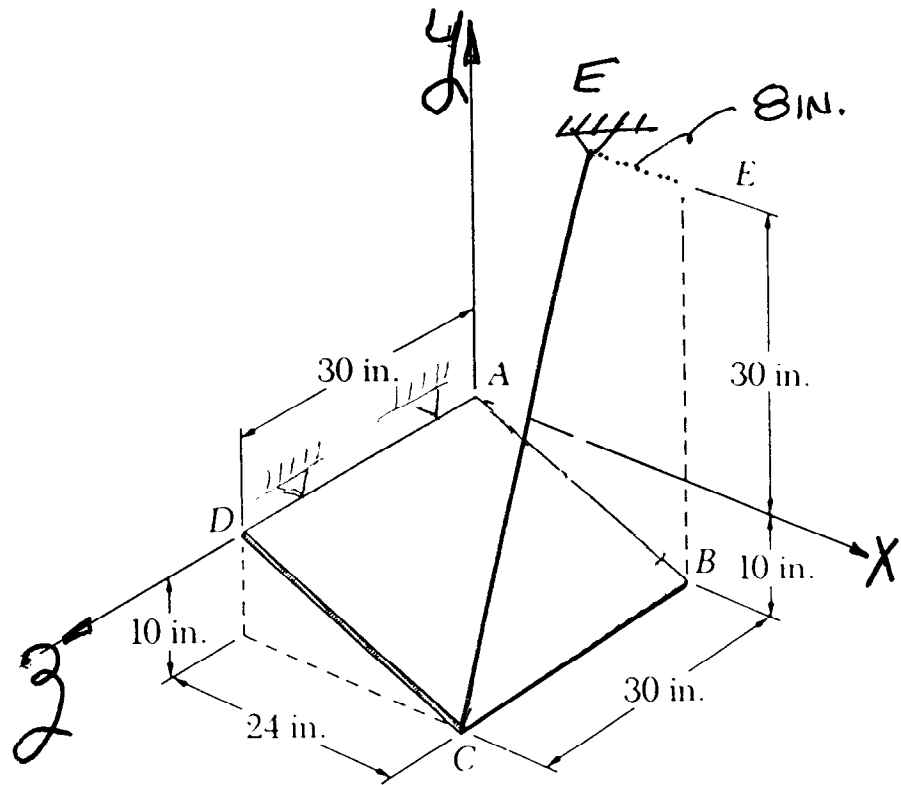
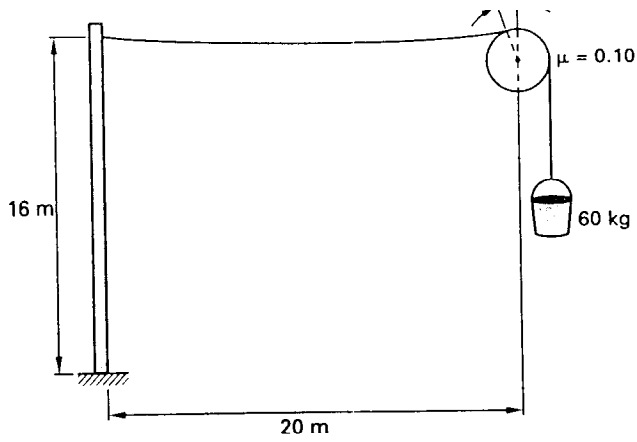


Fig. P4.99





- (A) 0
- (B) 10.0 kg
- (C) 11.6 kg
- (D) 12.1 kg

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$$= -2000 \text{ N} + (1000 \text{ N})(\sin 45^\circ) + W \sin \beta$$

$$\frac{W \sin \beta}{W \cos \beta} = \tan \beta = \frac{2000 \text{ N} - (1000 \text{ N})(\sin 45^\circ)}{(1000 \text{ N})(\cos 45^\circ)}$$

$$= 1.828$$

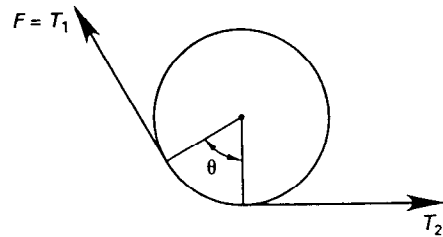
$$\beta = \tan^{-1}(1.828) = 61.3^\circ$$

$$W = \frac{(1000 \text{ N})(\cos 45^\circ)}{\cos 61.3^\circ}$$

$$= 1472 \text{ N} \quad (1500 \text{ N})$$

Answer is C.

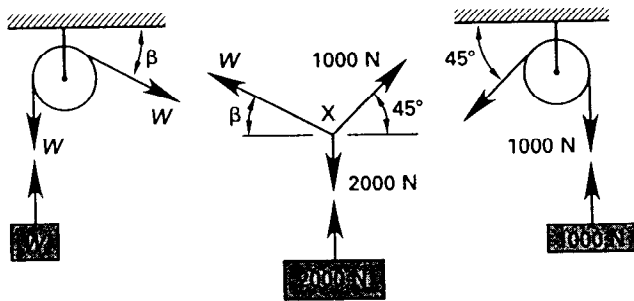
Solution 2:



SOLUTIONS TO FE-STYLE EXAM PROBLEMS

Solution 1:

The free-body diagrams are



$$F = T_1$$

$$T_2 = \mu_1 N = \mu_1 mg$$

$$= (0.58)(150 \text{ kg}) \left(9.81 \frac{\text{m}}{\text{s}^2} \right)$$

$$= 853.5 \text{ N}$$

$$\theta = (60^\circ) \left(\frac{2\pi \text{ rad}}{360^\circ} \right) = 1.0472 \text{ rad}$$

$$T_1 = T_2 e^{\mu\theta}$$

$$= (853.5 \text{ N}) e^{(0.9)(1.0472)}$$

$$= 2190 \text{ N} \quad (2200 \text{ N})$$

Answer is C.