Review for
Fundamentals of Engineering Exam

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Fluid Mechanics

- Fluid Properties
- Fluid Statics
- Fluid Dynamics
- Fluid Measurements

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Topic VII: Fluid Mechanics

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states of matter

- solid
- fluid
  - liquid
  - gas

A fluid is a substance that deforms continuously under the action of an applied shear stress.

Fluid Properties

density
specific volume
specific weight
specific gravity
pressure
stress
viscosity
surface tension
capillarity

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Mass and Force Units

dimension: <u>mass</u>   <u>force</u>

Systeme International (SI) units:

kilogram (kg) newton (N)

British Gravitational System units:

slug   pound

English Engineering System units:

pound mass (lbm) pound force (lbf)

Newton's Second Law:

force = mass $\cdot$ acceleration

$$F = m \cdot a$$

weight = mass $\cdot$ acceleration of gravity

$$W = m \cdot g$$

standard acceleration of gravity

$$g = 9.807 \text{ m/s}^2 = 32.174 \text{ ft/s}^2$$

SI System

$$w = m\,g$$

$$1\text{N} = 1\text{Kg·m/s}^2$$

British Gravitational System

$$m = w / g$$

$$1 \text{ slug} = \text{lb} / (\text{ft/s}^2) = \text{lb·s}^2/\text{ft}$$

English Engineering System

$$F = m\,a / g_c$$

where: $g_c = 32.174$ ft·lbm/lbf·s^2

$$1\ lbf = \frac{1\ lbm \times 32.174\ ft/s^2}{32.174\ ft \times lbm\ /\ lbf \times s^2}$$

Density (ρ) and Specific Weight (γ)

ρ = mass / unit volume

$$(kg/m^3, \text{slugs/ft}^3, \text{lbm/ft}^3)$$

γ = weight / unit volume

$$(N/m^3, \text{lb/ft}^3, \text{lbf/ft}^3)$$

SI and British Gravitational System:

$$\gamma = \rho g$$

English Engineering System:

$$\gamma = \rho (g / g_c)$$

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For water at temperature

of 10°C or 50°F:

ρ = 1,000 kg/m^3

= 1.94 slugs/ft^3

= 62.4 lbm/ft^3

γ = 9.80 kN/m^3

= 62.4 lb/ft^3

= 62.4 lbf/ft^3

Specific Gravity (S.G.)

The specific gravity of a fluid is the ratio

of the density of the fluid to the density

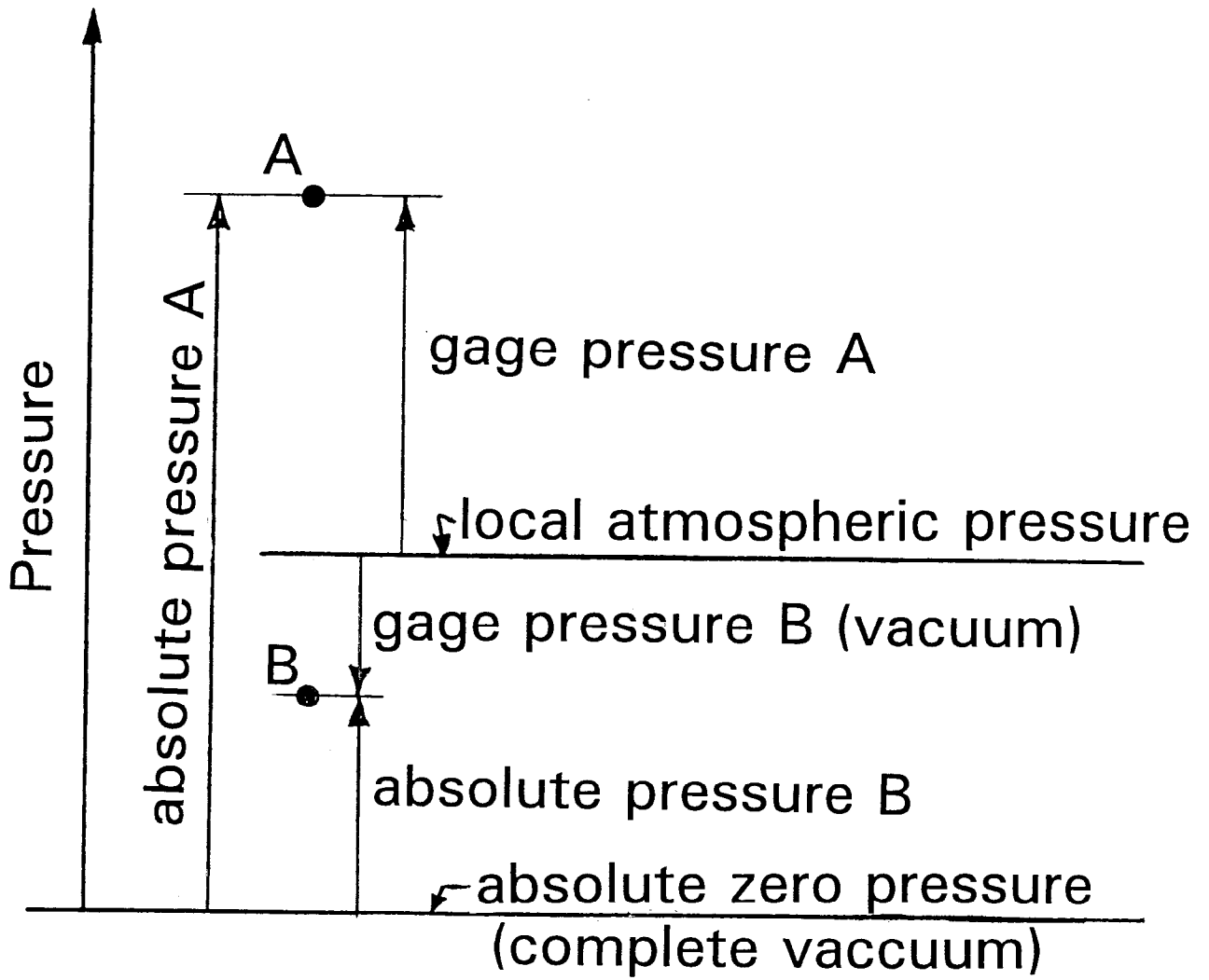of water at a specified temperature and

pressure.

$$S.G. = \rho_{fluid} / \rho_{water}$$

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Absolute and Gage Pressure

absolute pressure =

gage pressure + atmospheric pressure

$$P_{abs} = P_{gage} + P_{atm}$$

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Pressure

A

gage pressure A

local atmospheric pressure

gage pressure B (vacuum)

B

absolute pressure A

absolute pressure B

absolute zero pressure
(complete vaccuum)

Stress

stress (τ) is force per unit area

- normal stress
- tangential (shear) stress

Viscosity

The viscosity of a fluid is a measure of its resistance to flow when acted upon by an external force such as a pressure gradient or gravity.

Newton's Law of Viscosity

$$\tau = \mu \, (dv/dy)$$

τ - shear stress

dv/dy - velocity gradient

μ - absolute or dynamic
viscosity

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$$\nu \ - \ \mu/\rho$$

ν - kinematic viscosity
$$m^2/s \ \ or \ \ ft^2/s$$

μ - dynamic viscosity
$$N{\cdot}s/m^2 \ \ or \ \ lb{\cdot}s/ft^2$$

ρ - density
$$kg/m^3 \ \ or \ \ slug \ \ or \ \ lbm/ft^3$$

For water at temperature of

of 10°C or 50°F:

$$\mu = 1.307 \times 10^{-3} \text{ N·s/m}^2$$

$$= 2.735 \times 10^{-5} \text{ lb·s/ft}^2$$

$$v = 1.306 \times 10^{-6} \text{ m}^2/\text{s}$$

$$= 1.410 \times 10^{-5} \text{ ft}^2/\text{s}$$

Surface Tension

Surface tension is property used to describe a phenomena observed at the interface between a gas and liquid where intermolecular cohensive forces form an imaginary film capable of resisting tension.
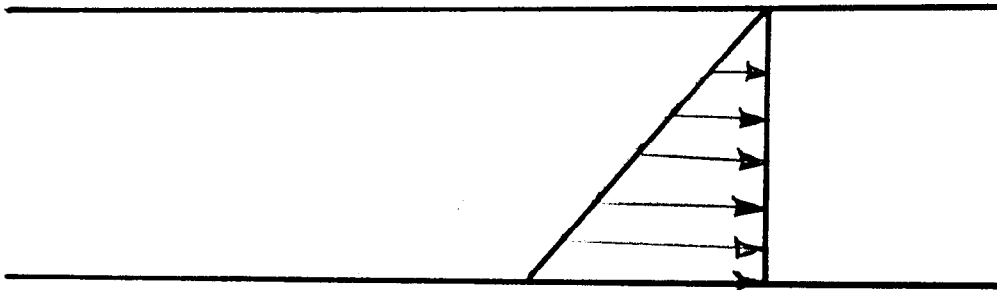
Capillarity

Capillary action is caused by surface tension between a liquid and solid surface. Water rises in a thin-bore tube.

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Fluid Statics

Hydrostatic pressure is the pressure
a fluid exerts on an immersed
object or on container walls.

hydrostatic pressure distribution

Fluid Statics

$$P_1 - P_2 = \gamma(z_2 - z_1)$$

$$P = \gamma h$$

$$F = pA$$

where:

P_1 = pressure at elevation z_1
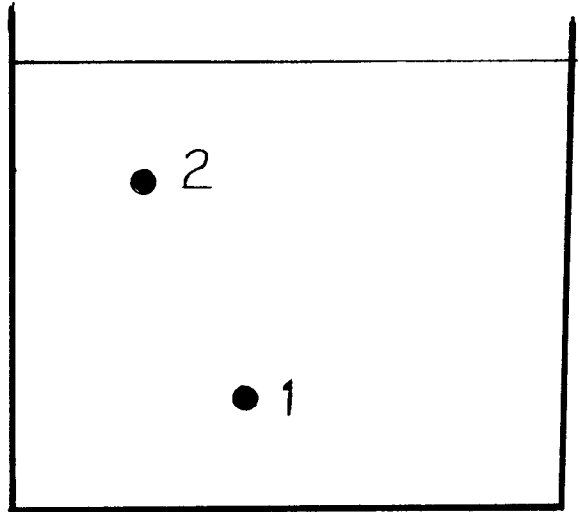
P_2 = pressure at elevation z_2

h = $z_2 - z_1$ = head

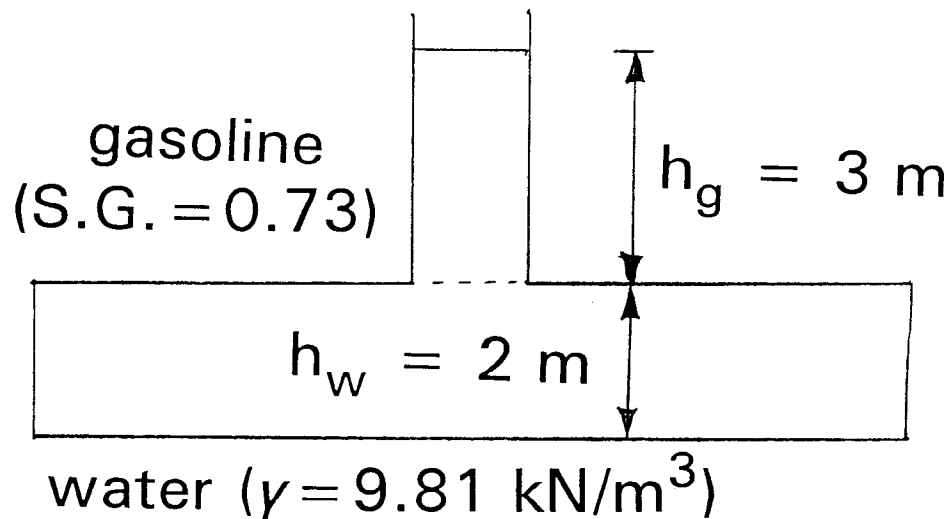p = pressure for head h

γ = specific weight

F = force

A = area

Example: What is the pressure 10 feet below the surface of a swimming pool?

$$p = \gamma h = (62.4 \text{ lb/ft}^3)(10\text{ft})$$

$$= 624 \text{ lb/ft}^2$$

Example: The tank of water has a 3-m

column of gasoline (S.G. = 0.73) above

it. Atmospheric pressure is 101 kPa.
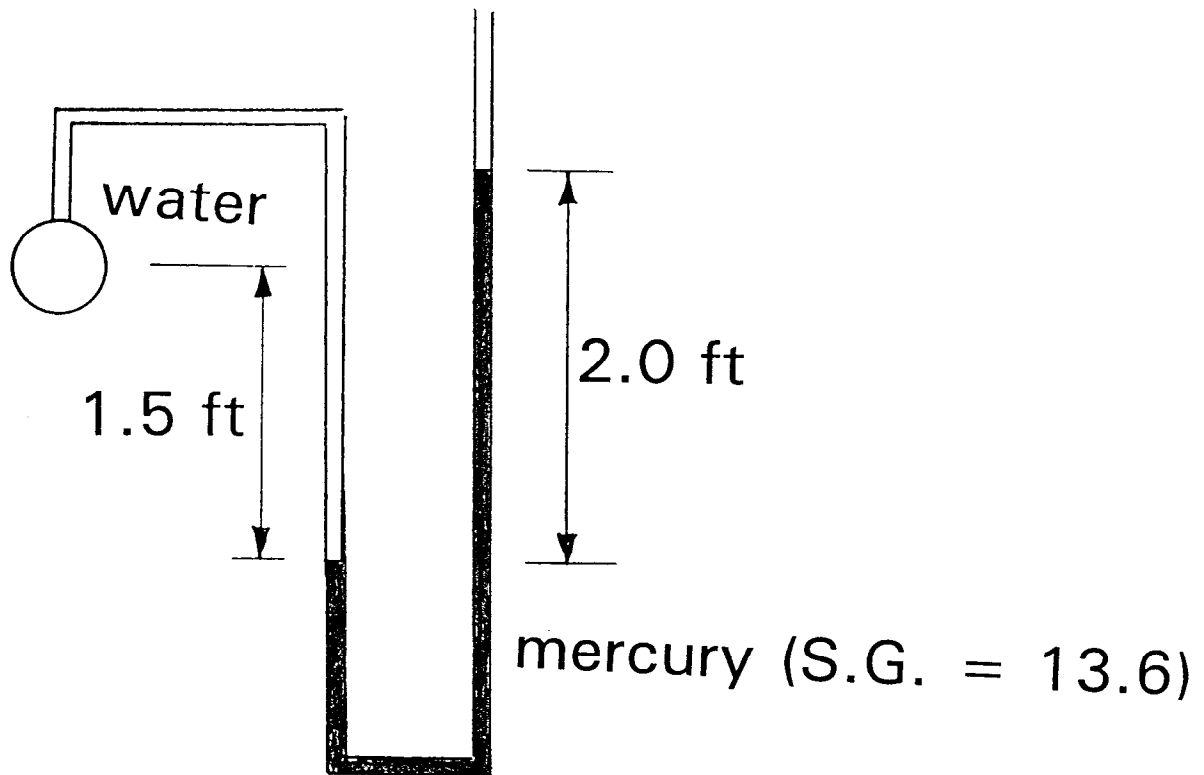
Compute the pressure on the bottom of

the tank.

gasoline
(S.G. = 0.73)

h_g = 3 m

h_w = 2 m

water (γ = 9.81 kN/m^3)

pressure at bottom of tank

$$P_{gage} = \gamma_w h_w + \gamma_g h_g$$

$$= (9.81 \text{ kN/m}^3)(2m)$$

$$+ (0.73)(9.81 \text{ kN/m}^3)(3m)$$

$$= 41.1 \text{ kN/m}^2$$

$$P_{abs} = P_{gage} + P_{atm}$$

$$= 41 \text{ kN/m}^2 + 101 \text{ kN/m}^2$$

$$= 142 \text{ kN/m}^2$$

$$= 142 \text{ kPa}$$

Example: Use the manometer

measurements to compute the pressure

in the pipe.

water

1.5 ft

2.0 ft

mercury (S.G. = 13.6)

$$P_{pipe} + \gamma_w h_w - \gamma_m h_m = 0$$

$$P_{pipe} + (62.4 \ lb/ft^3)(1.5ft)$$

$$- (13.6)(62.4 \ lb/ft^3)(2.0 \ ft) = 0$$

$$P_{pipe} = 1,604 \ lb/ft^2$$

Forces on Submerged Surfaces

Plane Surface

$$F = \gamma h_c A \qquad \text{magnitude}$$

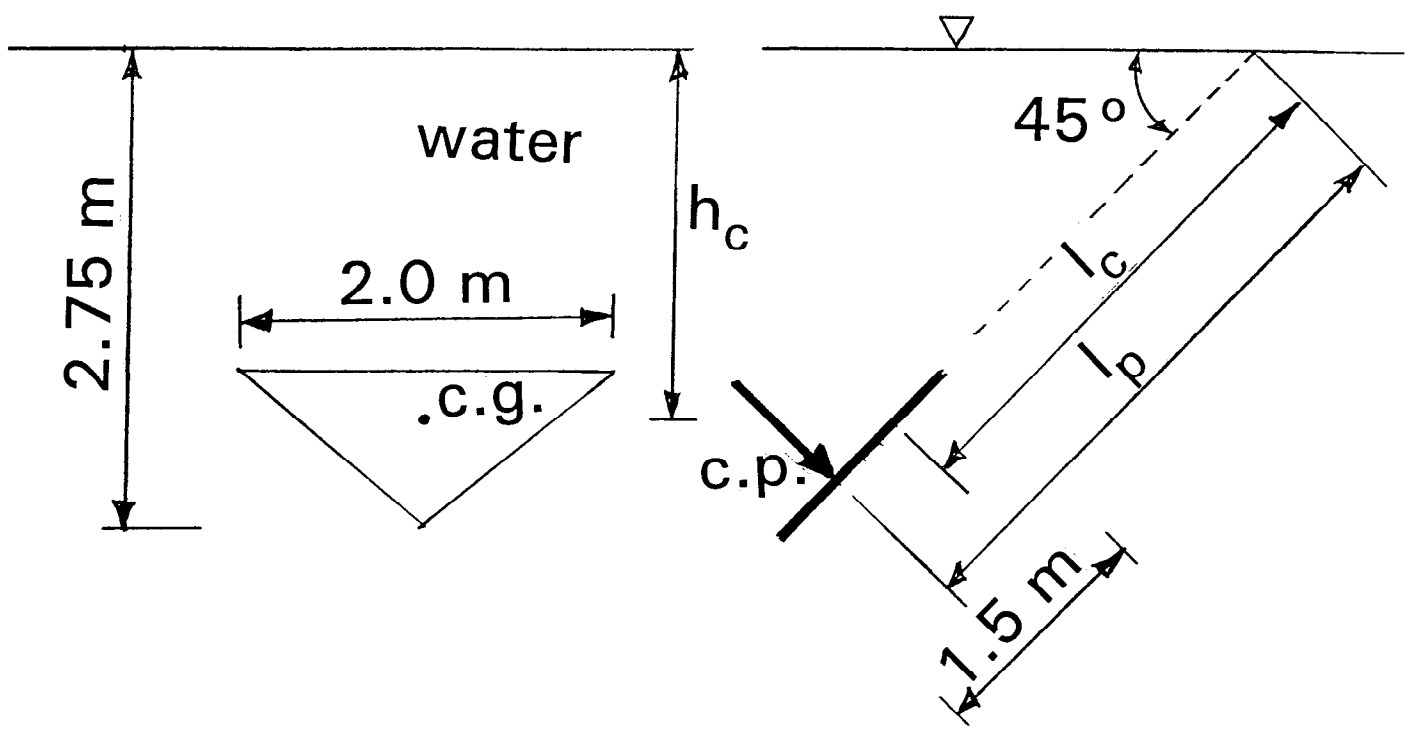$$I_p - I_c = I_c / (I_c A) \quad \text{location}$$

Curved or Plane Surface

$$F_h = \gamma h_c A \qquad \text{(vertical projection)}$$

$$F_v = \gamma V \qquad \text{(weight of fluid)}$$

Buoyant Force

$$F = \gamma \text{ (volume displaced)}$$

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Example: Compute the magnitude and

location of the resultant force.

moment of inertia $I_c = bh^3/36$

$$F = \gamma h_c A$$

$$I_p - I_c = I_c / (I_c A)$$

$$A = 0.5 \ (2m)(1.5m) = 1.5 \ m^2$$

$$h_c = 2.75 \ m - [\ (2/3) \ (1.5m) \ ] \sin 45°$$

$$= 2.043 \ m$$

$$I_c = h_c / \sin 45° = 2.043 \ m / 0.7071$$

$$= 2.889 \ m$$

moment of inertia $I_c = bh^3/36$

$I_c = (2m)(1.5m)^3/36 = 0.1875\ m^4$

$F = \gamma h_c A$

$\quad = (9.80\ kN/m^3)(2.043\ m)(1.5\ m^2)$

$\quad = 30.0\ kN$

$I_p - I_c = I_c / (I_c A)$

$\quad = 0.1875\ m^4/[(2.889m)\ (1.5m^2)]$

$\quad = 0.0433\ m$

$I_p = 0.0433\ m + 2.889\ m = 2.932\ m$

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Example: Compute the force on the curved corner for a unit width.

water

10 m

3 m

3 m

quarter circle
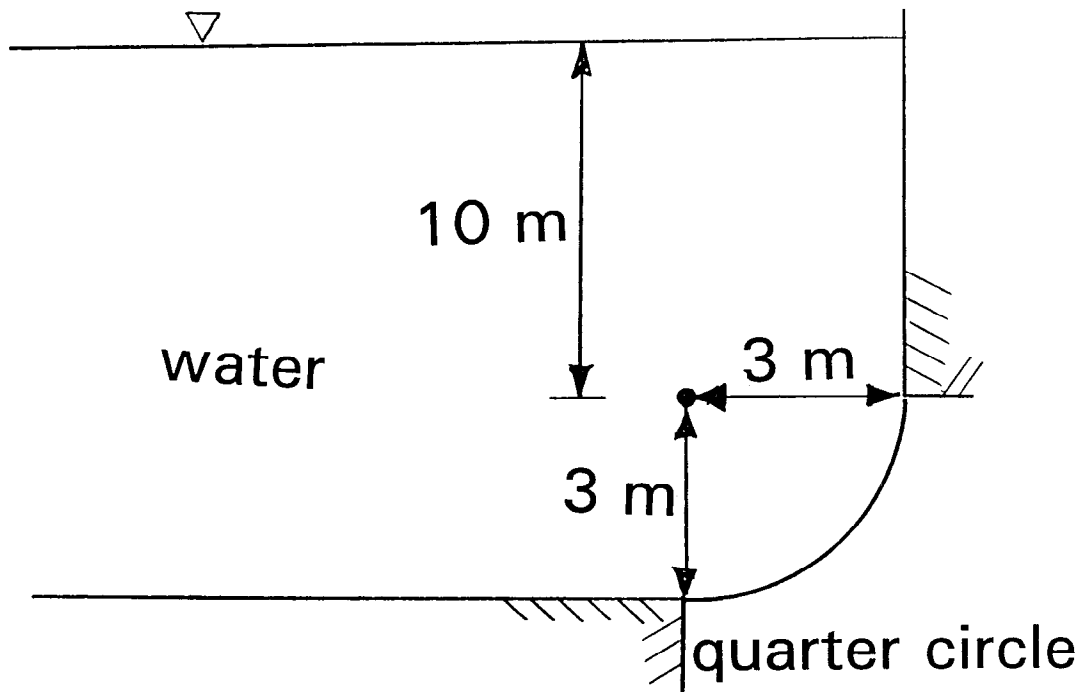
$$F_H = \gamma h_c A$$

$$= (9.80 \text{ kN/m}^3)\,(11.5\text{m})\,(3\text{m}^2)$$

$$= 338 \text{ kN}$$

$$F_v = \gamma V$$

volume $(V) =$

$$(10\text{m})\,(3\text{m})\,(1\text{m}) + (1/4)\pi\,(3\text{m})^2\,(1\text{m})$$
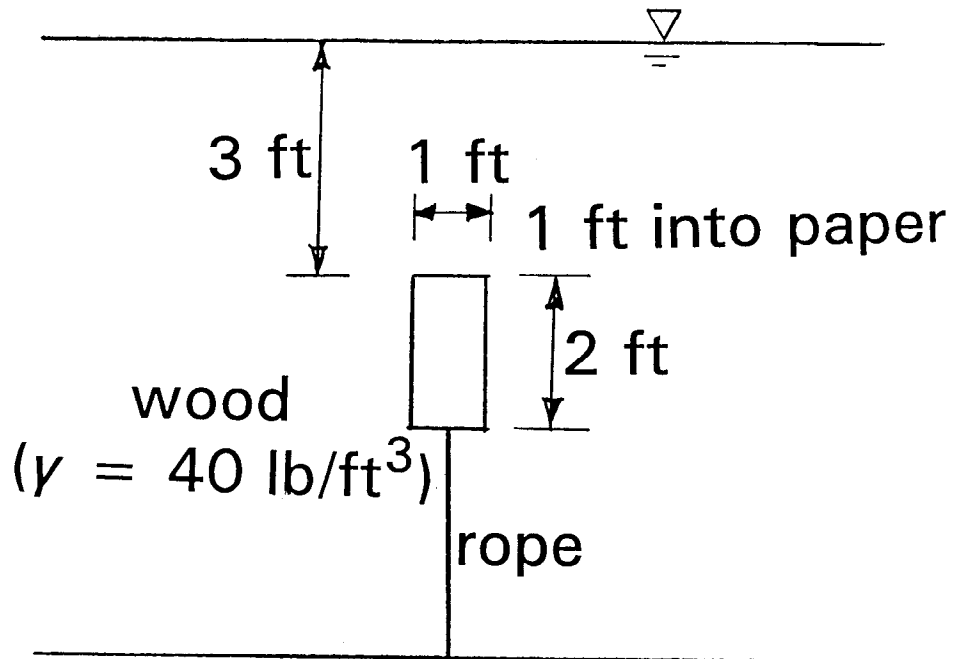
$$= 37.07 \text{ m}^3$$

$$F_v = (9.80 \text{ kN/m}^3)(37.07 \text{ m}^3) = 363 \text{ kN}$$

$$F = (F_H^{\,2} + F_V^{\,2})^{0.5} = (338^2 + 363^2)^{0.5}$$

$$= 496 \text{ kN}$$

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Laws of Buoyancy and Flotation

I.   A body immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced.

2.   A floating body displaces its own weight of the liquid in which it floats.

Example: Compute the force in the rope.

3 ft

1 ft

1 ft into paper

2 ft

wood
(γ = 40 lb/ft³)

rope

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buoyant force (F_B) =

γ (volume displaced)

F_B = $(62.4 \text{ lbs/ft}^3) (2 \text{ ft}^3)$

= 124.8 lbs

weight of wood (W)

$W = (40 \text{ lbs/ft}^3) (2 \text{ ft}^3) = 80 \text{ lbs}$

$F_B - W - F_{rope} = 0$

124.8 lbs - 80lbs - F_{rope} = 0

F_{rope} = 44.8 lbs

Alternative Solution

The buoyant force (F_B) on a submerged

body is the difference between the

vertical component of pressure force on

its underside and upper side.

$$F = \gamma h_c A \quad \text{or} \quad F = \gamma V$$

$$F_B = (62.4 \text{ lb/ft}^3)(5 \text{ ft})(1 \text{ ft}^2)$$

$$- (62.4 \text{ lb/ft}^3)(3 \text{ ft})(1 \text{ ft}^2)$$

$$= 124.8$$

Fluid Dynamics

Conservation Laws
mass

energy

momentum

Flow in pressure conduits
Flow in open channels

Conservation of Mass

(Continuity Equation)

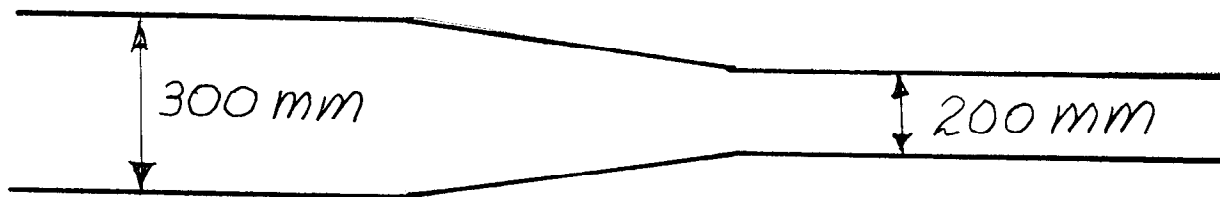$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For incompressible fluids ($\rho_1 = \rho_2$)

$$A_1 V_1 = A_2 V_2$$

$$Q_1 = Q_2$$

Example: Three kN/s of water flows through the pipeline reducer. Determine the flow rate and velocity in the 300 mm and 200 mm pipes.

300 mm

200 mm

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$$\gamma_1 = \gamma_2 = 9.8 \text{ kN/m}^3$$

$$\gamma = \rho g$$

$$w = mg$$

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\dot{w} = \gamma_1 A_1 V_1 = \gamma_2 A_2 V_2$$

$$Q = \dot{w}/\gamma = 3 \text{ kN/s} / 9.8 \text{ kN/m}^3$$

$$Q = 0.306 \text{ m}^3/\text{s}$$

$$V = Q/A$$

$$A = 0.25\pi D^2$$

$$V_1 = 0.306 \text{ m}^3/\text{s} / 0.25\pi(0.3)^2$$

$$V_1 = 4.33 \text{ m/s}$$

$$V_2 = 0.306 \text{ m}^3/\text{s} / 0.25\pi(0.2)^2$$

$$V_2 = 9.74 \text{ m/s}$$
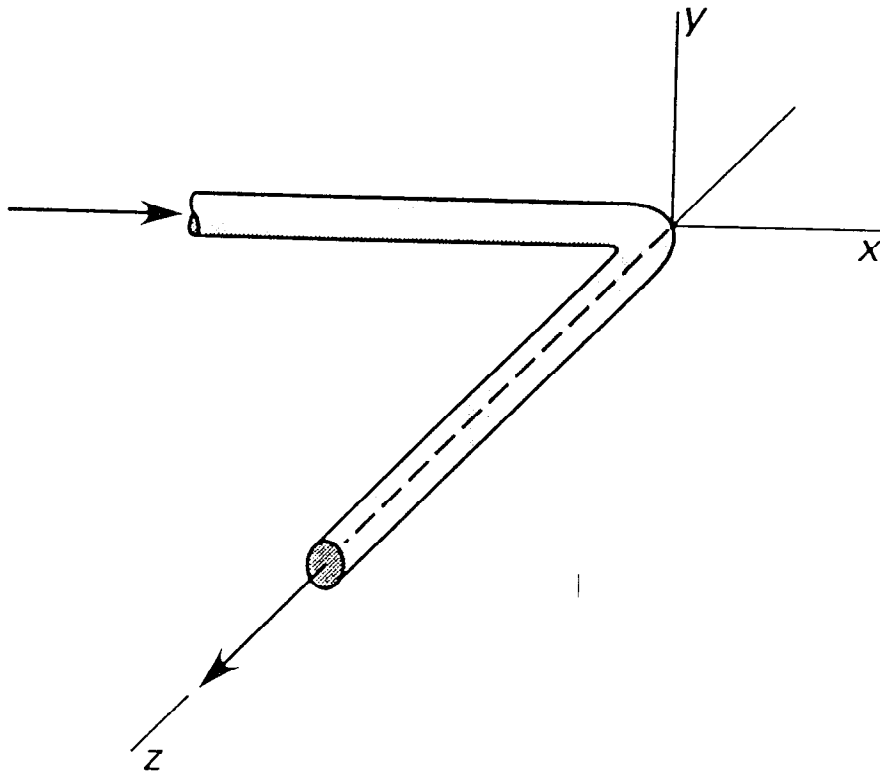
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Conservation of Momentum

(Impulse-Momentum Equation)

$$\Sigma \vec{F} = \rho Q (\vec{V}_{out} - \vec{V}_{in})$$

$$\Sigma F_x = \rho Q(V_{2x} - V_{1x})$$
$$\Sigma F_y = \rho Q(V_{2y} - V_{1y})$$
$$\Sigma F_z = \rho Q(V_{2z} - V_{1z})$$

Example: Water is flowing at 0.884 m^3/s through a 15 cm diameter pipe, that has a 90° bend. What is the reaction on the water in the z-direction in the bend?

$$\rho = 1000 \text{ kg/m}^3$$

$$V = Q/A$$
$$A = 0.25\pi D^2$$
$$V = 0.884 \text{ m}^3/\text{s} / 0.25\pi(0.15 \text{ m})^2$$
$$V = 50 \text{ m/s}$$

$$\sum F_z = \rho Q(V_{2z} - V_{1z})$$
$$F_z = (1000 \text{ kg/m}^3)(0.884 \text{ m}^3/\text{s})(0\text{-}50 \text{ m/s})$$
$$F_z = -44{,}200 \text{ kg·m/s}$$
$$F_z = -44{,}200 \text{ N}$$

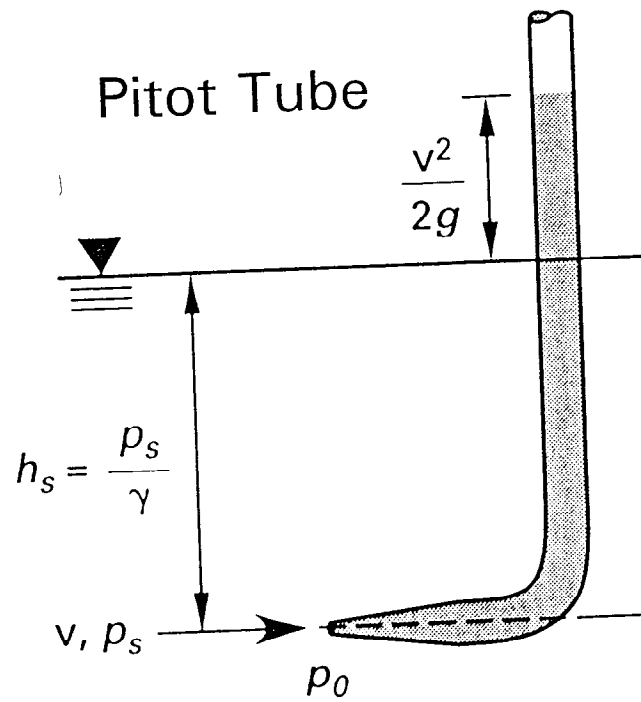$$\text{Reaction} = 44{,}200 \text{ N} = 44.2 \text{ kN}$$

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Conservation of Energy

(Bernoulli and Energy Equations)

$$total\ head = z + \frac{P}{\gamma} + \frac{V^2}{2g}$$

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_p = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

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Pitot Tube

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

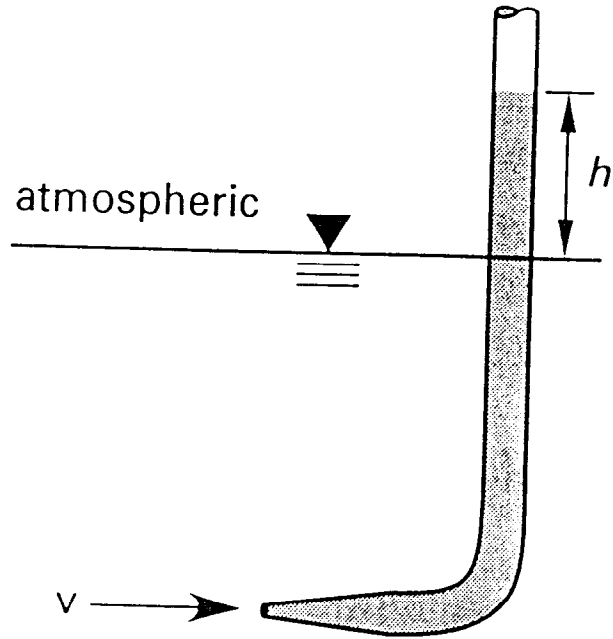$$0 + \frac{P_0}{\gamma} + 0 = 0 + \frac{P_S}{\gamma} + \frac{V^2}{2g}$$

$$\frac{P_0}{\gamma} - \frac{P_S}{\gamma} = \frac{V^2}{2g}$$

$$\frac{P_0}{\gamma} - \frac{P_S}{\gamma} = h$$
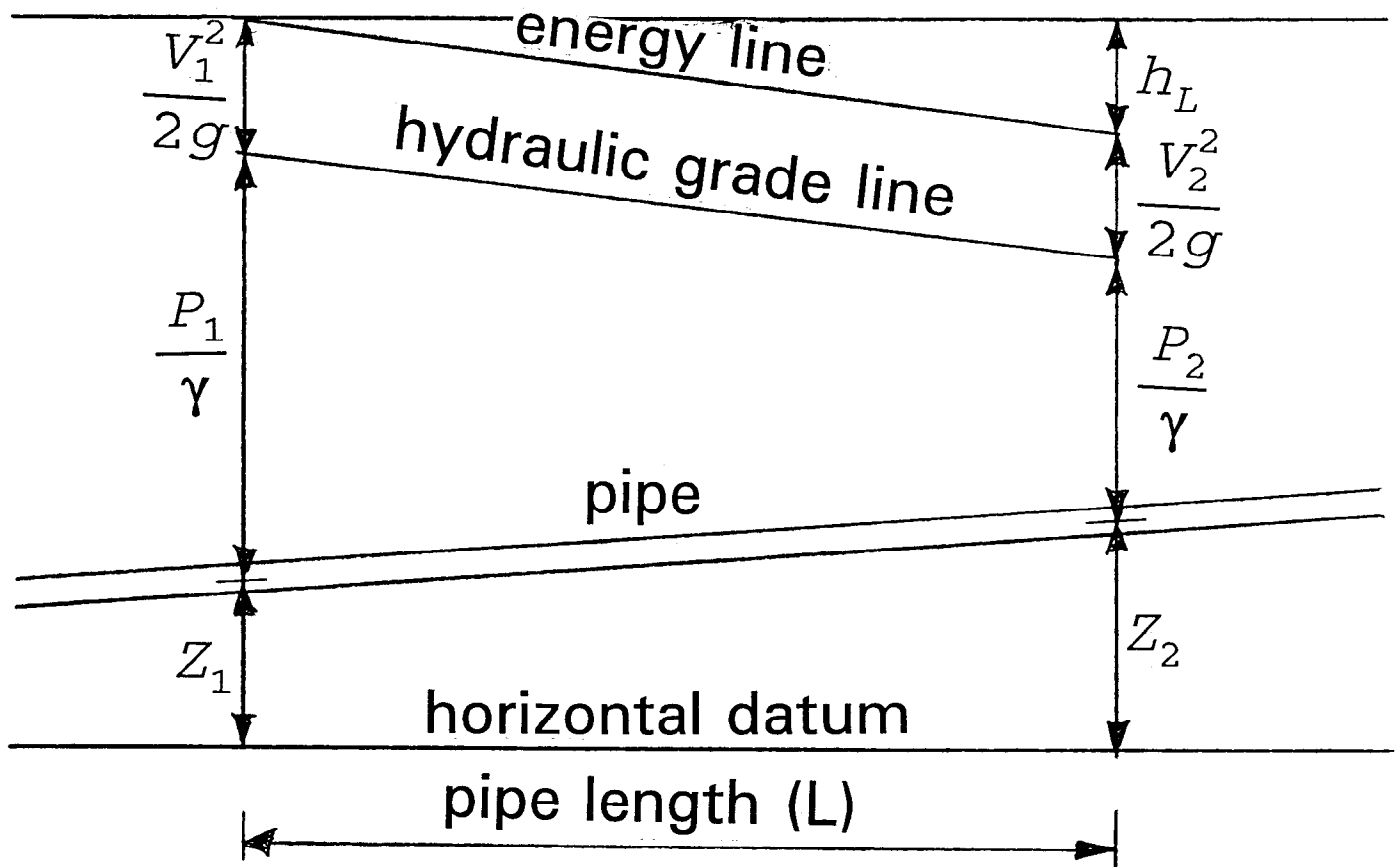
$$h = \frac{V^2}{2g}$$

$$V = (2gh)^{0.5}$$

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Example: The height of water in the pitot tube is measured to be 7.3 cm. What is the velocity at that point in the flow.

$$V = (2gh)^{0.5}$$
$$V = [2(9.81 \text{ m/s}^2)(0.073 \text{ m})]^{0.5}$$
$$V = 1.2 \text{ m/s}$$

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slope of energy line S = h_L/L

head = energy per unit weight

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

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Pressure and Pressure Head
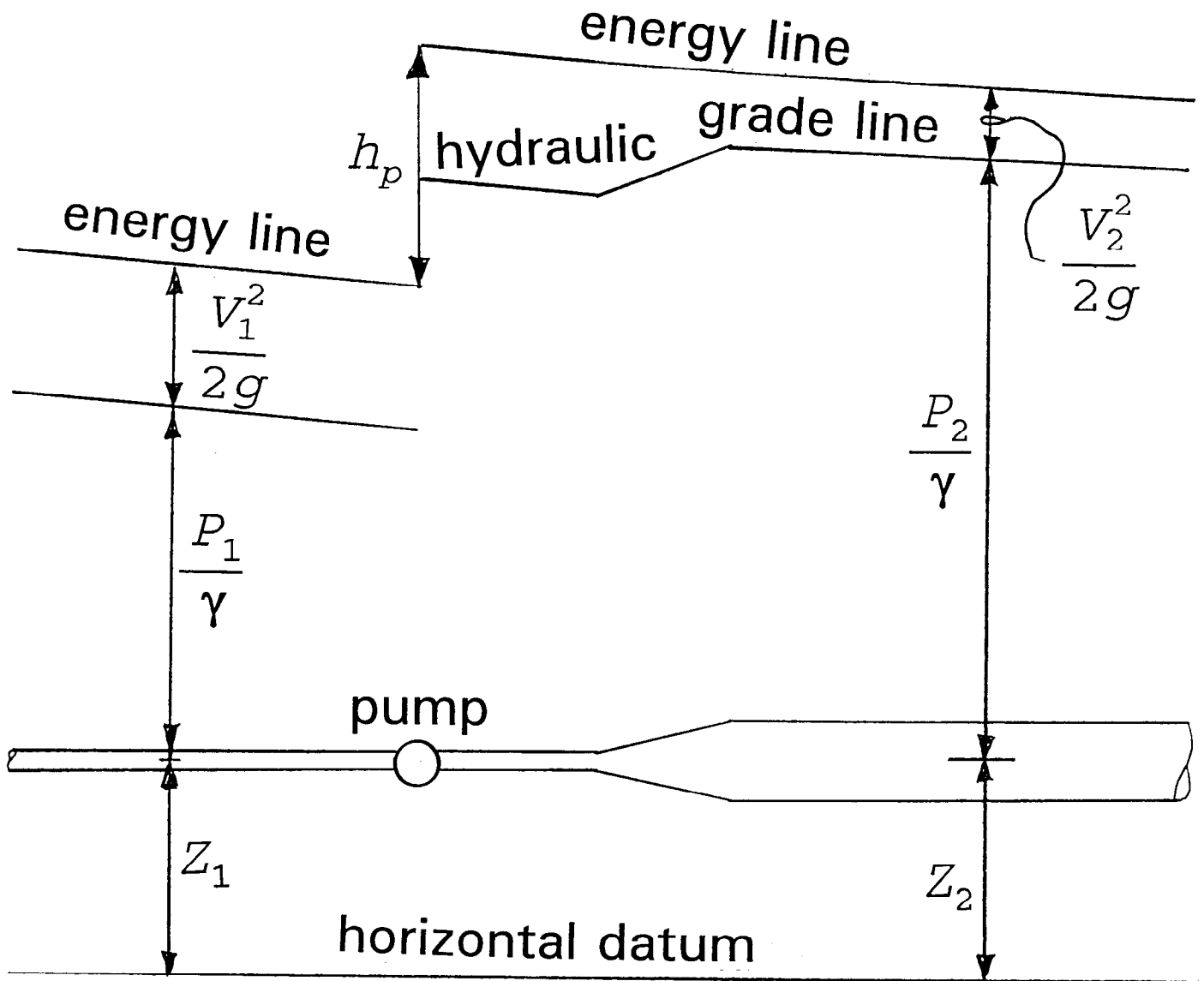
$$p = \gamma h \qquad h = p/\gamma$$

for $p = 1$ psi (lb/inch^2)

$$h = (1\ \text{lb/in}^2)\ (144\ \text{in}^2/\text{ft}^2)\ /\ 62.4\ \text{lb/ft}^3$$

$$= 2.31\ \text{ft}$$

Pressures in municipal water distribution

systems are typically 60 - 80 psi

(139 - 185 ft).

faucet pressures > 5 psi (11.5 ft)

main pressures > 35 psi (80.8 ft)

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energy line

grade line

h_p hydraulic

$\dfrac{V_2^2}{2g}$

energy line

$\dfrac{V_1^2}{2g}$

$\dfrac{P_1}{\gamma}$

$\dfrac{P_2}{\gamma}$

pump

Z_1

Z_2

horizontal datum

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_p = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

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Darcy - Weisbach Equation

$$h_L = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

h_L = head loss due to pipe friction

(feet or meters)

f = friction factor (dimensionless)

L = length of pipe (ft or m)

D = pipe diameter (ft or m)

V = flow velocity (ft/sec or m/sec)

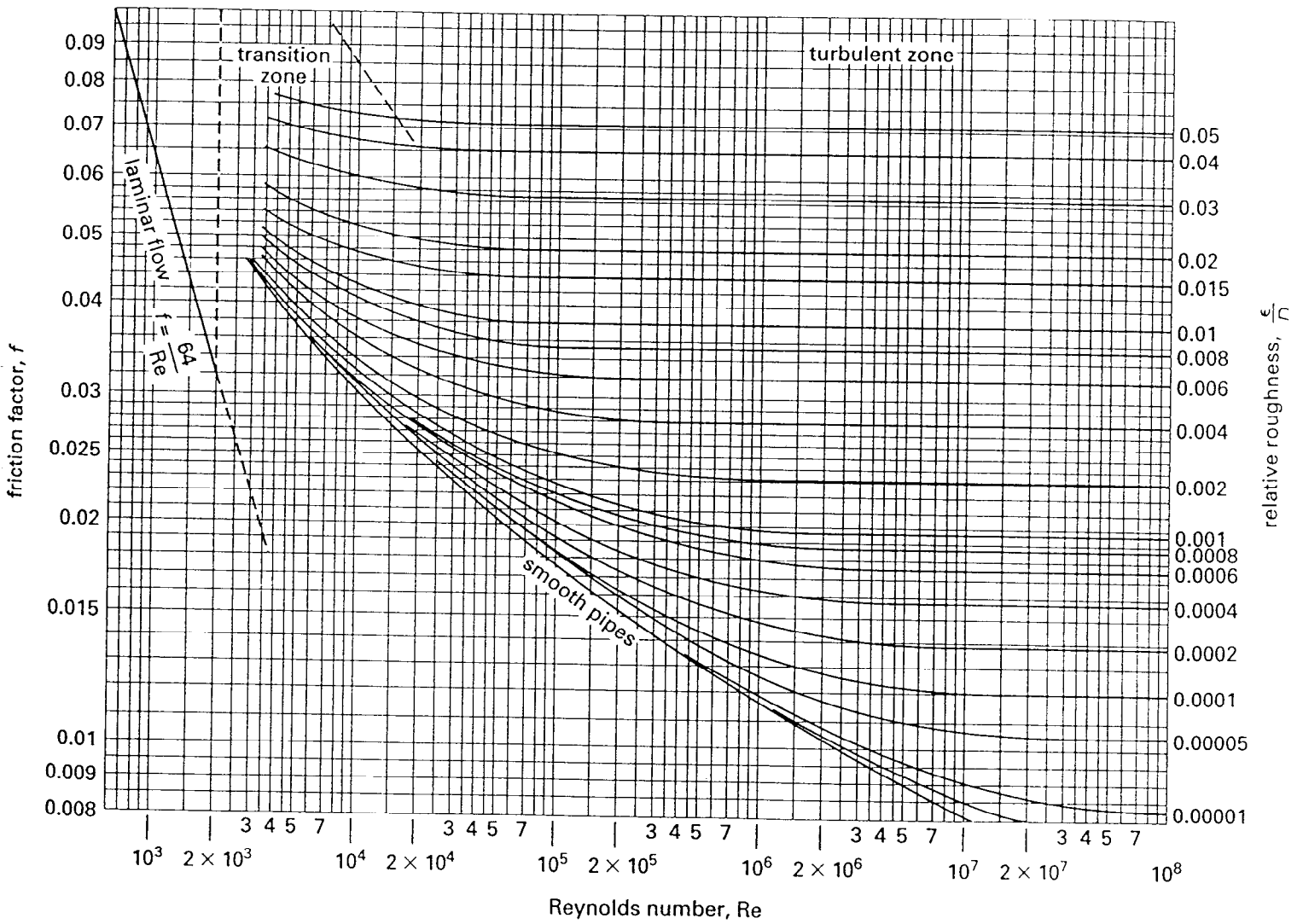g = gravitational acceleration constant

(32.2 ft/sec^2 or 9.81 m/sec^2)

ϵ = roughness (ft or m)

ϵ/D = relative roughness (dimensionless)

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$$h_f = \frac{fLv^2}{2Dg}$$

material	ϵ	
	ft	m
riveted steel	0.003–0.03	0.9–9.0
concrete	0.001–0.01	0.3–3.0
galvanized iron	0.00085	0.25
commercial steel		
or wrought iron	0.00015	0.046
drawn tubing	0.000005	0.0015

Figure 24.6 Moody Friction Factor Chart

$$\text{Re} = \frac{vD}{\nu}$$

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Darcy-Weisbach Example: For a discharge (Q) of 1.0 ft³/s, compute the head loss in 1,500 feet of new 6-inch diameter cast iron pipe (ϵ = 0.00085 ft).   Assume a water temperature of 60° F.

A = 1/4 π D² = 1/4 π (6/12)² = 0.196 ft²

V = Q/A = 1.0 ft³/s /0.196 ft² = 5.09 ft/s

v = 1.217 x 10⁻⁵ ft²/s

$$N_R = \frac{VD}{\upsilon} = \frac{(5.09 \; ft/s)(0.5ft)}{1.217 \cdot 10^5 \; ft^2/s} = 2.09 \bullet 10^5$$

ϵ/D = 0.00085 ft / 0.5 ft = 0.0017

f = 0.023

$$h_L \;\; = \;\; f \; \frac{L}{D} \; \frac{V^2}{2g}$$

$$h_L \;\; = \;\; 0.023 \; (\frac{1,500ft}{0.5ft}) \; \frac{(5.09ft/s)^2}{2(32.2ft/s^2)}$$

= 27.8 ft

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Minor Losses

$$h_L = C \left(V^2/2g\right)$$

exit/entrance condition	C value
exit, sharp	1.0
exit, protruding	0.8
entrance, sharp	0.5
entrance, rounded	0.1
entrance, gradual, smooth	0.04

<u>Example</u>: A 6-inch diameter 500 ft long
steel pipe (ϵ = 0.00015 ft) conveys flow
between two reservoirs which have a
difference in water surface elevation of
30 ft. The pipe exit and entrance are
square edge. Compute the flow rate.
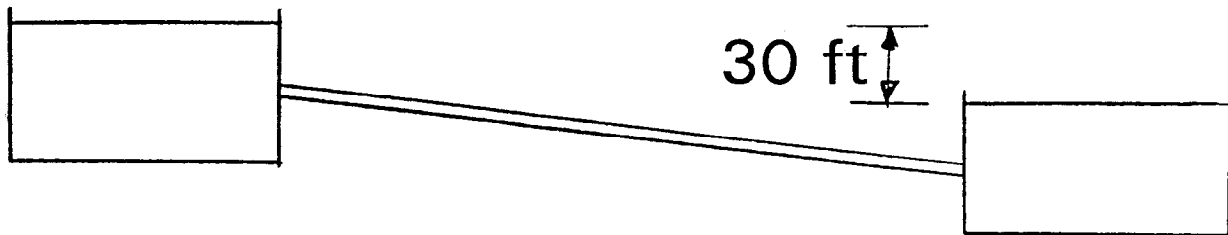
$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$30 + 0 + 0 = 0 + 0 + 0 + h_L$$

$$h_L = 30 \text{ ft}$$

Pipe Friction

ϵ/D = 0.00015 ft / 0.5 ft = 0.0003

assume f = 0.016

$$h_L = f \left(\frac{L}{D}\right) \frac{V^2}{2g}$$

$$h_L = 0.016 \left(\frac{500}{0.5}\right) \frac{V^2}{2g} = 16 \frac{V^2}{2g}$$

Minor Losses (entrance and exit)

$$h_{L_m} = 0.5 \frac{V^2}{2g} + 1.0 \frac{V^2}{2g} = 1.5 \frac{V^2}{2g}$$

Compute V for assumed f

$$h_L = 16 \frac{V^2}{2g} + 1.5 \frac{V^2}{2g} = 17.5 \frac{V^2}{2g} = 30 \; ft$$

$$V = 10.5 \; ft/s$$

Check f
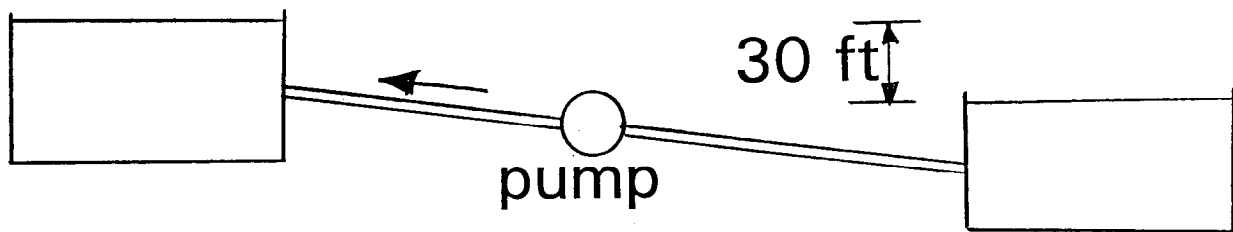
$$N_R = \frac{VD}{\nu} = \frac{(10.5 \ ft/s) \ (0.5 \ ft)}{1.22 \cdot 10^{-5} \ ft^2/s}$$

$$= 4.3 \times 10^5$$

$$f = 0.0164$$

Compute Q

$$Q = VA = (10.5 \ ft/s) \ (0.196 \ ft^2)$$

$$= 2.06 \ ft^3/s$$

Example: - Assume a pump is added to the previous example and the flow direction is reversed. What pump head is required for a discharge of 2.0 cfs.

$$A = 1/4\,\pi\,D^2 = 1/4\,\pi\,(0.5\text{ ft})^2$$
$$= 0.1963\text{ ft}^2$$

$$V = Q/A = 2\text{ cfs} / 0.1963\text{ ft}^2$$
$$= 10.2\text{ ft/s}$$

from previous problem:

$$h_L = 16\,\frac{V^2}{2g} + 1.5\,\frac{V^2}{2g} = 17.5\,\frac{V^2}{2g}$$

$$h_L = 17.5\,(\frac{(10.2\text{ ft/s})^2}{2(32.2\text{ ft/s}^2)}) = 28.2\text{ ft}$$

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_p = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$0 + 0 + 0 + hp = 30 + 0 + 0 + 28.2$$

$$hp = 58.2\text{ feet}$$

Power and Energy

energy - ability to do work

work - force through a distance

power - rate of transferring energy or
work per unit of time

power = energy/time =
(weight/time)(energy/weight)

$$P = \gamma \, Q \, h$$

units of power

kilowatt (kw) - kN·m/s
horsepower (hp) - 550 ft·lb/s
1 hp = 0.746 kw

$$P = \gamma \, Q \, h \, / \, \eta$$

P - power (kw = kN·m/s or ft·lb/s)

γ - unit weight (kN/m^3 or lb/ft^3)

Q - discharge (m^3/s or ft^3/s)

h - head (m or ft)

η - efficiency

brake horsepower - input horsepower
delivered to the pump shaft

$$BHP = \frac{\gamma Q h_p}{550 \eta}$$

horsepower (hp) - 550 ft·lb/s

<u>Example</u>: Determine the horsepower required for the pump of the previous problem, assuming the pump efficiency is 75%.

$$Q = 2.0 \text{ ft}^3/\text{s}$$

$$h_p = 58.2 \text{ ft}$$

$$\eta = 0.75$$

$$BHP = \frac{\gamma Q h_p}{550 \eta}$$

$$BHP = \frac{(62.4 \ lb/ft^3)(2.0 \ ft^3/s)(58.2 \ ft)}{(550 \ \frac{ft \bullet lb/s}{hp})(0.75)}$$

$$= 17.6 \text{ horsepower}$$

Example: Compute the discharge rate that causes the pressure to drop to vapor pressure. The pipe between the reservoir and pump has a length of 1,000 ft, diameter of 3 feet, and friction factor (f) of 0.02. Neglect minor losses. For a water temperature of 80°F , the vapor pressure (P_V) is 0.51 psia.
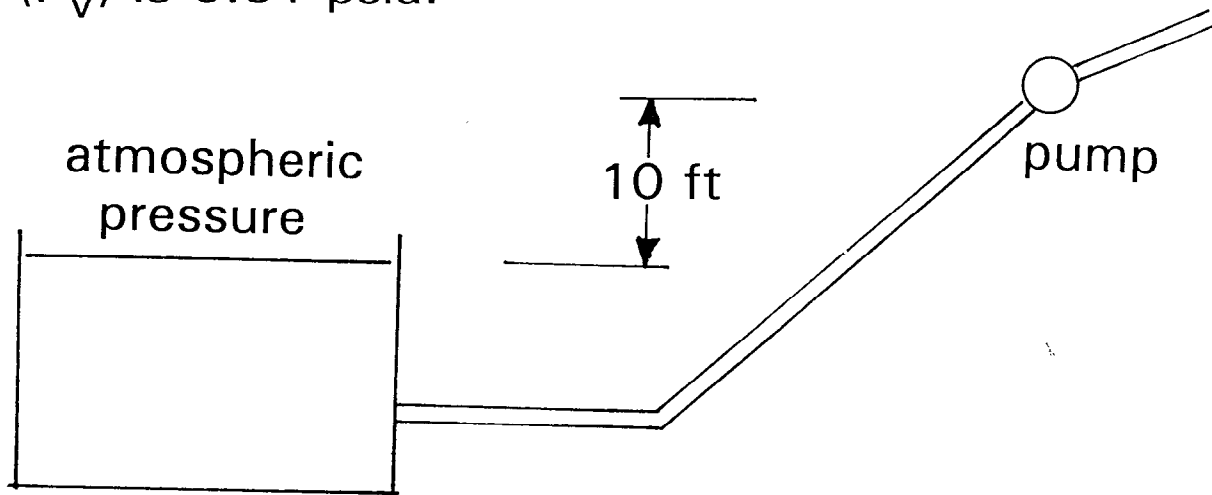
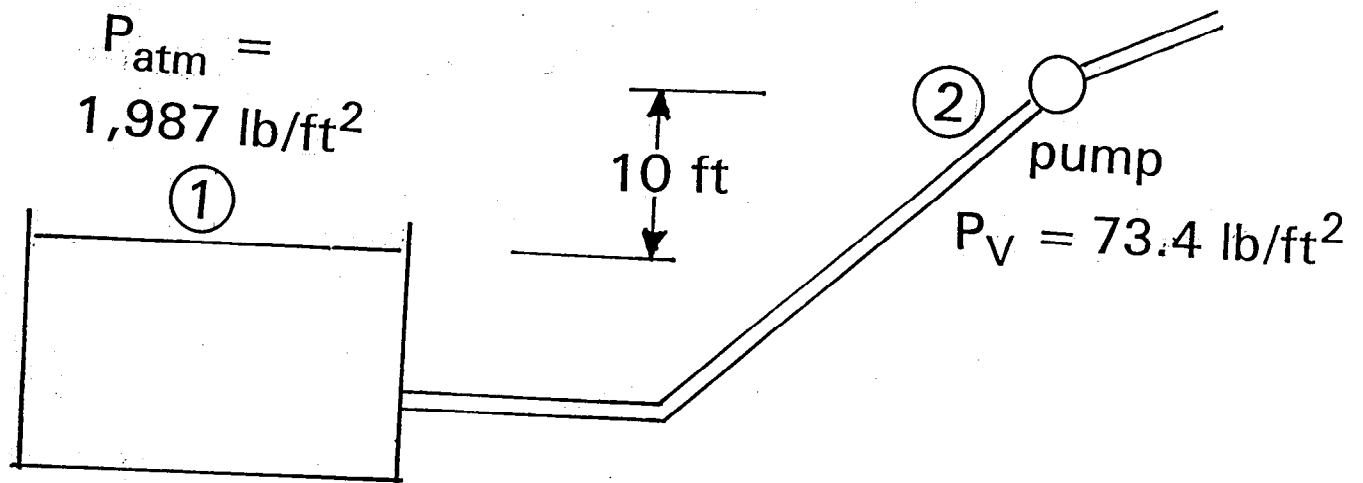$$P_{atm} = 13.8 \text{ psia} = 1,987 \text{ lb/ft}^2$$
$$P_V = 0.51 \text{ psia} = 73.4 \text{ lb/ft}^2$$
$$\gamma = 62.22 \text{ lb/ft}^3$$

$$h_L = f(L/D)\,(V^2/2g)$$
$$h_L = (0.02)(1000\text{ft}/3\text{ft})(V^2/[2(32.2 \text{ ft/s}^2)])$$
$$h_L = 0.119V^2$$

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

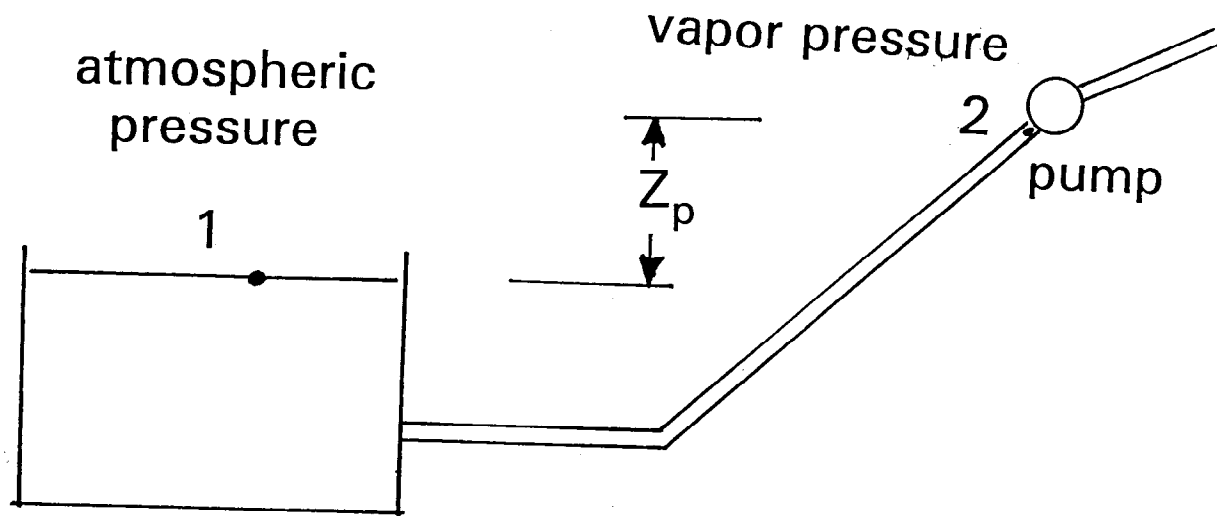$$0 + \frac{P_{atm}}{\gamma} + 0 = Z_P + \frac{P_V}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$\frac{1,987 \; lb/ft^2}{62.22 \; lb/ft^3} = 10 \, ft \; +$$

$$\frac{73.4 \; lb/ft^2}{62.22 \; lb/ft^3} + \frac{V^2}{2(32.2 \; ft/s^2)} + 0.119 \, V^2$$

V = 13.2 ft/s

Q = VA = (13.2 ft/s) (7.069 ft^2)
Q = 93.3 ft^3/s

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atmospheric pressure

vapor pressure

1

2

pump

Z_p

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$0 + \frac{P_{atm}}{\gamma} + 0 = Z_P + \frac{P_v}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$\frac{P_{atm} - P_v}{\gamma} = Z_p + \frac{V^2}{2g} + h_L$$

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$$h_L = KQ^N$$
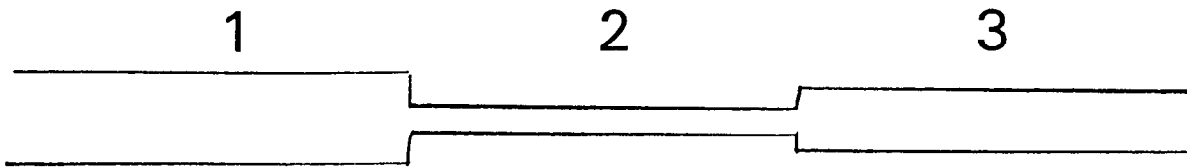$$h_L = KQ^2$$

Darcy Weisbach
$$h_f = f(L/D)(1/2A^2g)Q^2$$
$$K = f(L/D)(1/2A^2g)$$

Minor Losses
$$h_L = C(Q^2/2gA^2)$$
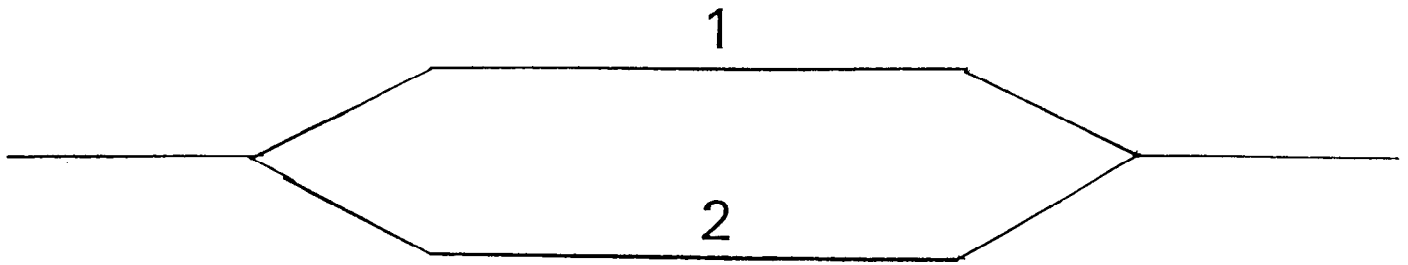$$K = C/2gA^2$$

FOR PIPES IN SERIES

$$Q \quad = \quad Q_1 \quad = \quad Q_2 \quad = \quad Q_3$$

$$h_L \quad = \quad h_{L1} \quad + \quad h_{L2} \quad + \quad h_{L3}$$

$$h_{LE} \quad = \quad h_L$$

$$K_E Q^n \quad = \quad K_1 Q^n \quad + \quad K_2 Q^n \quad + \quad K_3 Q^n$$

$$K_E \quad = \quad K_1 \quad + \quad K_2 \quad + \quad K_3$$
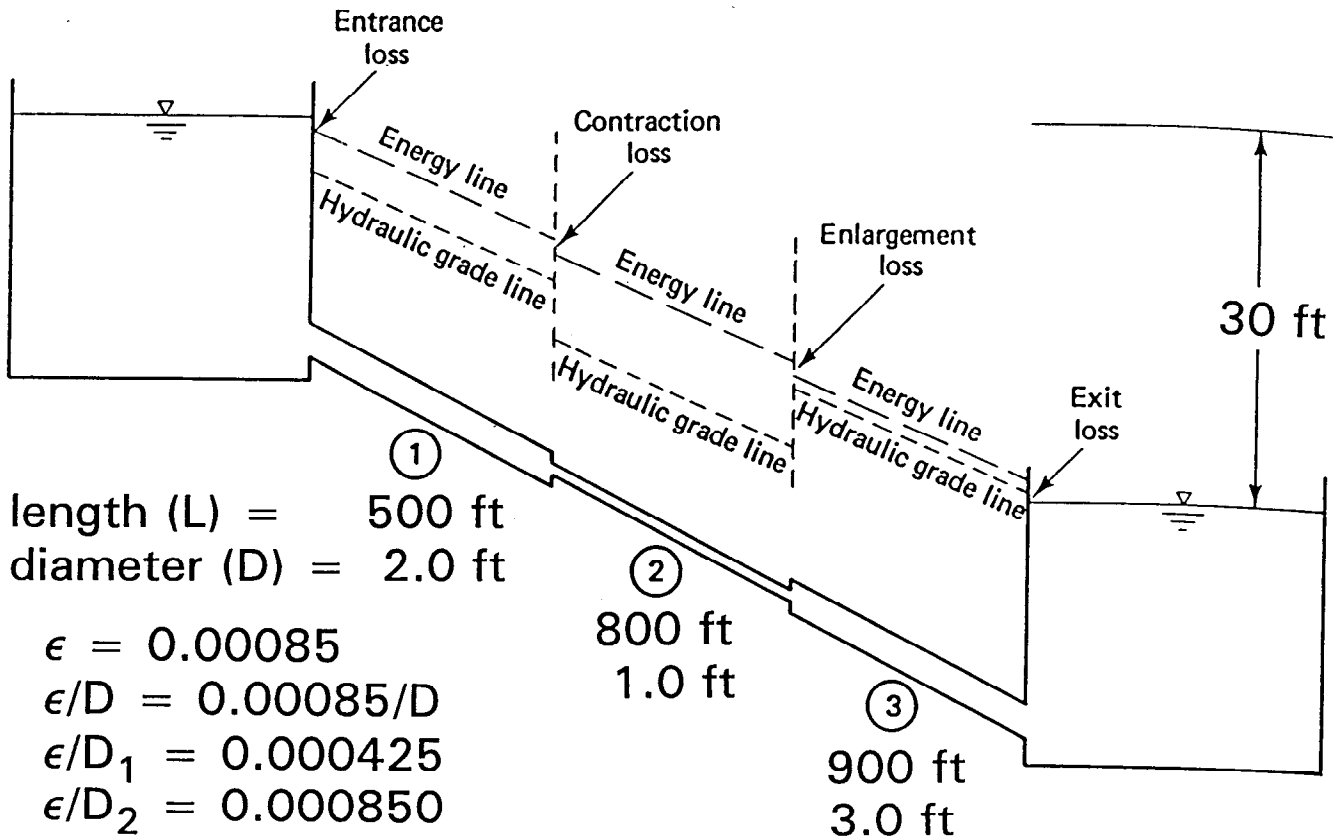
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FOR PIPES IN PARALLEL

$$Q = Q_1 + Q_2$$

$$h_L = h_{L1} = h_{L2}$$

$$h_L = KQ^n \qquad Q = (h_L/K)^{1/n}$$

$$(h_{LE}/K_E)^{1/n} = (h_{L1}/K_1)^{1/n} + (h_{L2}/K_2)^{1/n}$$

$$(1/K_E)^{1/n} = (1/K_1)^{1/n} + (1/K_2)^{1/n}$$

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Example: Compute the discharge.
Neglect the minor losses.

Entrance loss

Contraction loss

Enlargement loss

Exit loss

Energy line

Hydraulic grade line

30 ft

length (L) = 500 ft
diameter (D) = 2.0 ft

$\epsilon = 0.00085$
$\epsilon/D = 0.00085/D$
$\epsilon/D_1 = 0.000425$
$\epsilon/D_2 = 0.000850$
$\epsilon/D_3 = 0.000283$

800 ft
1.0 ft

900 ft
3.0 ft

estimated f
$f_1 = 0.0170$
$f_2 = 0.0190$
$f_3 = 0.0160$

$A = 1/4 \, \pi \, D^2$
$A_1 = 3.142 \text{ ft}^2$
$A_2 = 0.7854 \text{ ft}^2$
$A_3 = 7.069 \text{ ft}^2$

$$h_L = f(L/D)(Q^2/2gA^2) = KQ^2$$

$K = F(L/D)(Q^2/2gA^2)$
$K_1 = 0.00669$
$K_2 = 0.38259$
$K_3 = 0.001491$

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$$Q = Q_1 = Q_2 = Q_3$$

$$h_L = h_{L1} + h_{L2} + h_{L3} = 30 \text{ ft}$$

$$K_1 Q^2 + K_2 Q^2 + K_3 Q^2 = 30 \text{ ft}$$

$$0.00669 \, Q^2 + 0.38259 \, Q^2 + 0.001491 \, Q^2 = 30 \text{ ft}$$

$$Q = 8.76 \text{ cfs}$$

Check the original estimates of friction factor (f):

$V = Q/A$
$V_1 = 8.76 \text{ ft}^3/s \; / \; 3.142 \text{ ft}^2 = 2.79 \text{ ft/s}$
$V_2 = 8.76 \text{ ft}^3/s \; / \; 0.7854 \text{ ft}^2 = 11.15 \text{ ft/s}$
$V_3 = 8.76 \text{ ft}^3/s \; / \; 7.069 \text{ ft}^2 = 1.24 \text{ ft/s}$

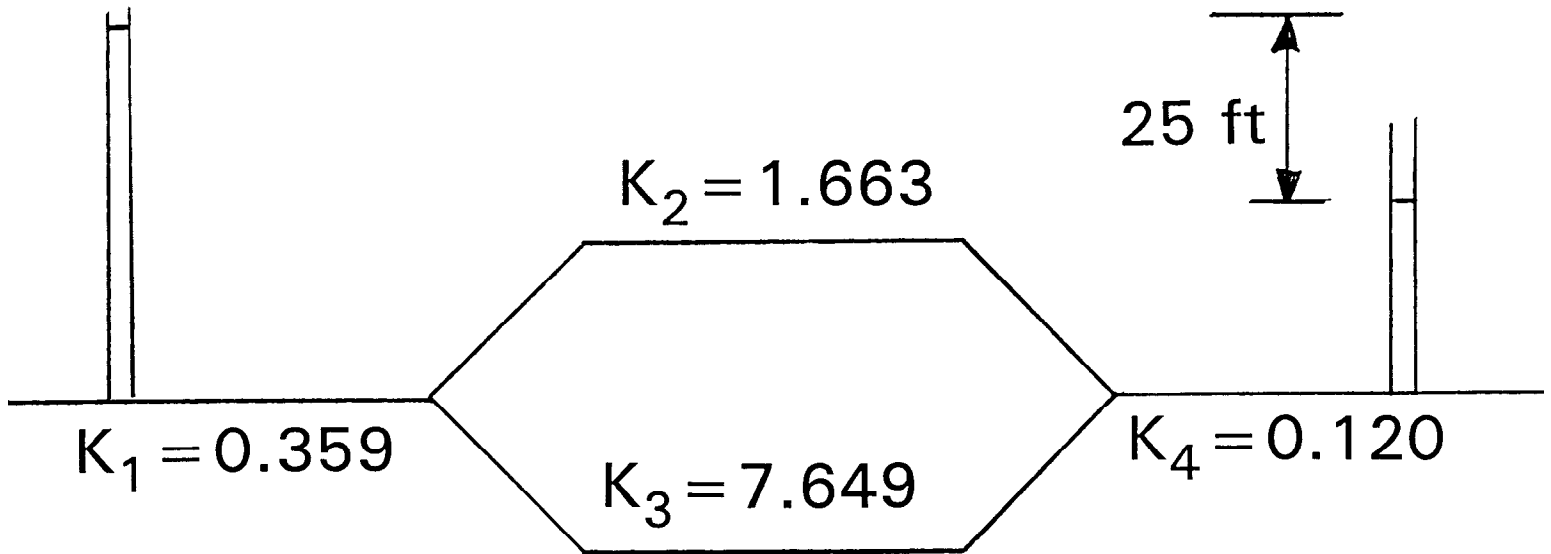$N_R = DV/v$
$N_R = 4.57 \times 10^5$
$N_R = 9.14 \times 10^5$
$N_R = 3.05 \times 10^5$

$f = .017$
$f = .019$
$f = .017$

Example: Compute the discharge in each pipe. Neglect minor losses.

$$h_L = KQ^2$$

$$K = f\left(\frac{L}{D}\right)\left(\frac{1}{2gA^2}\right)$$

Pipe	L(feet)	D(inch)	A(ft^2)	f	K=f(L/D)(1/2gA^2)
1	3,000	16	1.396	0.020	0.359
2	3,000	12	0.785	0.022	1.663
3	2,000	8	0.349	0.020	7.649
4	1,000	16	1.396	0.020	0.120

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$$h_{L2} = h_{L3}$$
$$K_2 Q_2{}^2 = K_3 Q_3{}^2$$
$$1.663 Q_2{}^2 = 7.649 Q_3{}^2$$
$$Q_2 = 2.145 Q^3$$

$$Q_1 = Q_2 + Q_3$$
$$Q_1 = 2.145 Q_3 + Q_3$$
$$Q_1 = 3.145 Q_3$$
$$Q_3 = 0.318 Q_3$$

$$h_L = h_{L1} + h_{L4} + h_{L3}$$

$$25 = K_1 Q_1{}^2 + K_4 Q_4{}^2 + K_3 Q_3{}^2$$

$$25 = K_1 Q_1{}^2 + K_4 Q_4{}^2 + K_3 (0.318 Q_1)^2$$
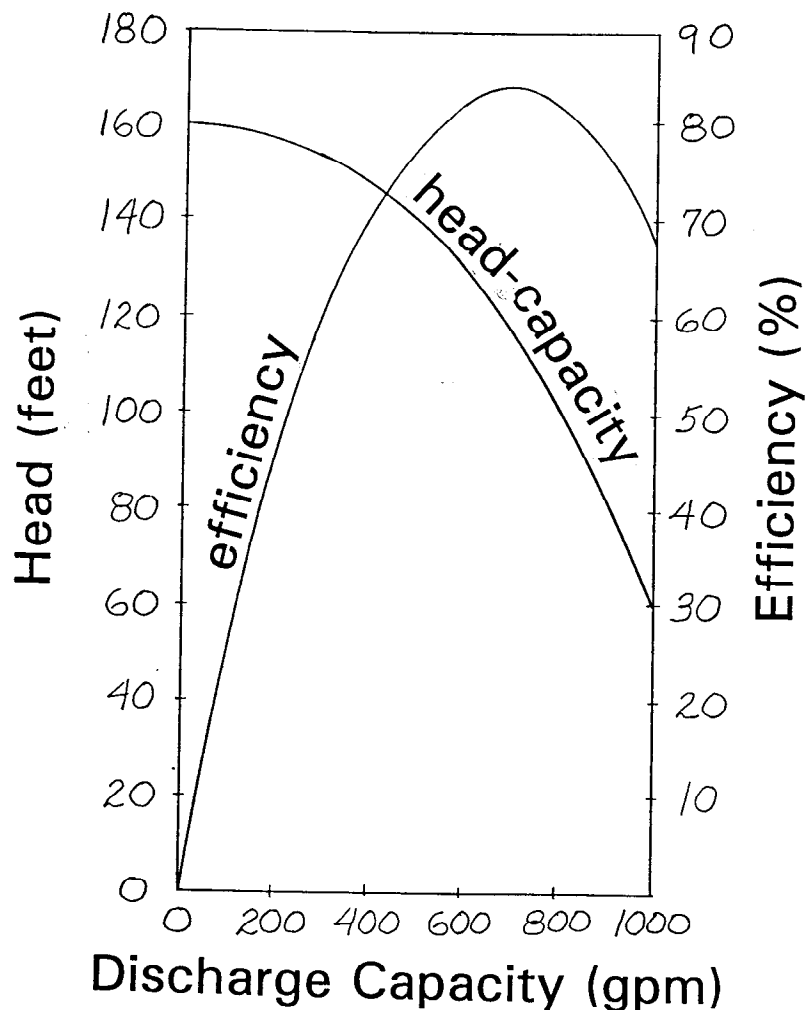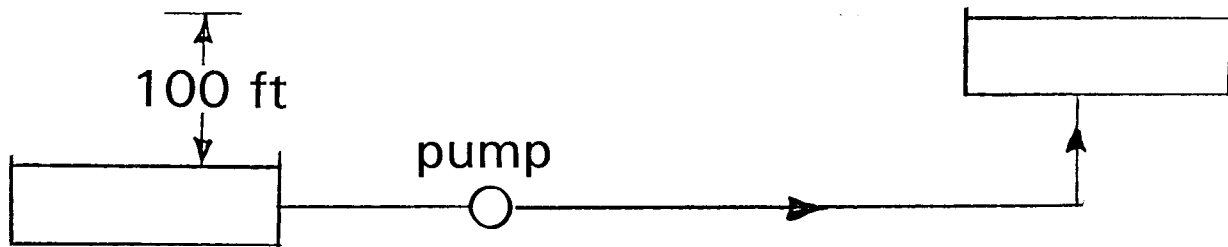
$$25 = 1.252 Q_1{}^2$$

$$Q_1 = 4.47 \text{ cfs}$$
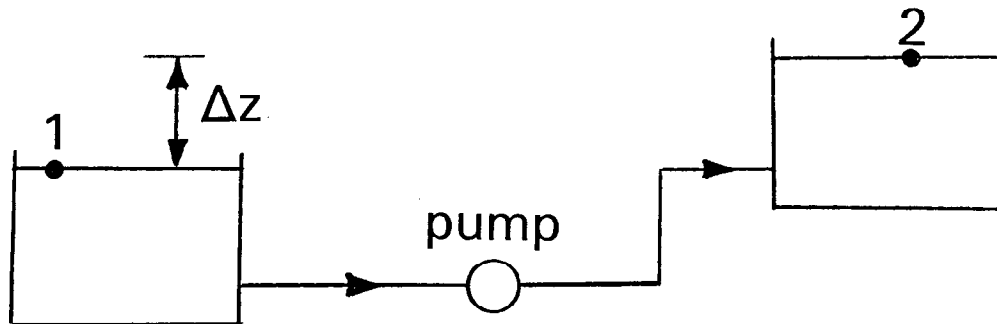
$$Q_4 = Q_1 = 4.47 \text{ cfs}$$

$$Q_3 = 0.318 Q_1 = 0.318 (4.47 \text{ cfs})$$
$$= 1.42 \text{ cfs}$$

$$Q_2 = 2.145 Q_3 = 2.145 (1.42 \text{ cfs})$$
$$= 3.05 \text{ cfs}$$

EXAMPLE: Two reservoirs are connected by a 850 feet long 6-inch diameter pipe (f = 0.020). A pump with the given characteristic curves is used to lift water from one reservoir to the other. Determine the discharge rate.

$$Z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + h_p = Z_2 + \frac{P_2}{\gamma} + \frac{V_1^2}{2g} + h_L$$

$$z_1 + 0 + 0 + h_p = z_2 + 0 + 0 + h_L$$

$$h_p = (z_2 - z_1) + h_L$$
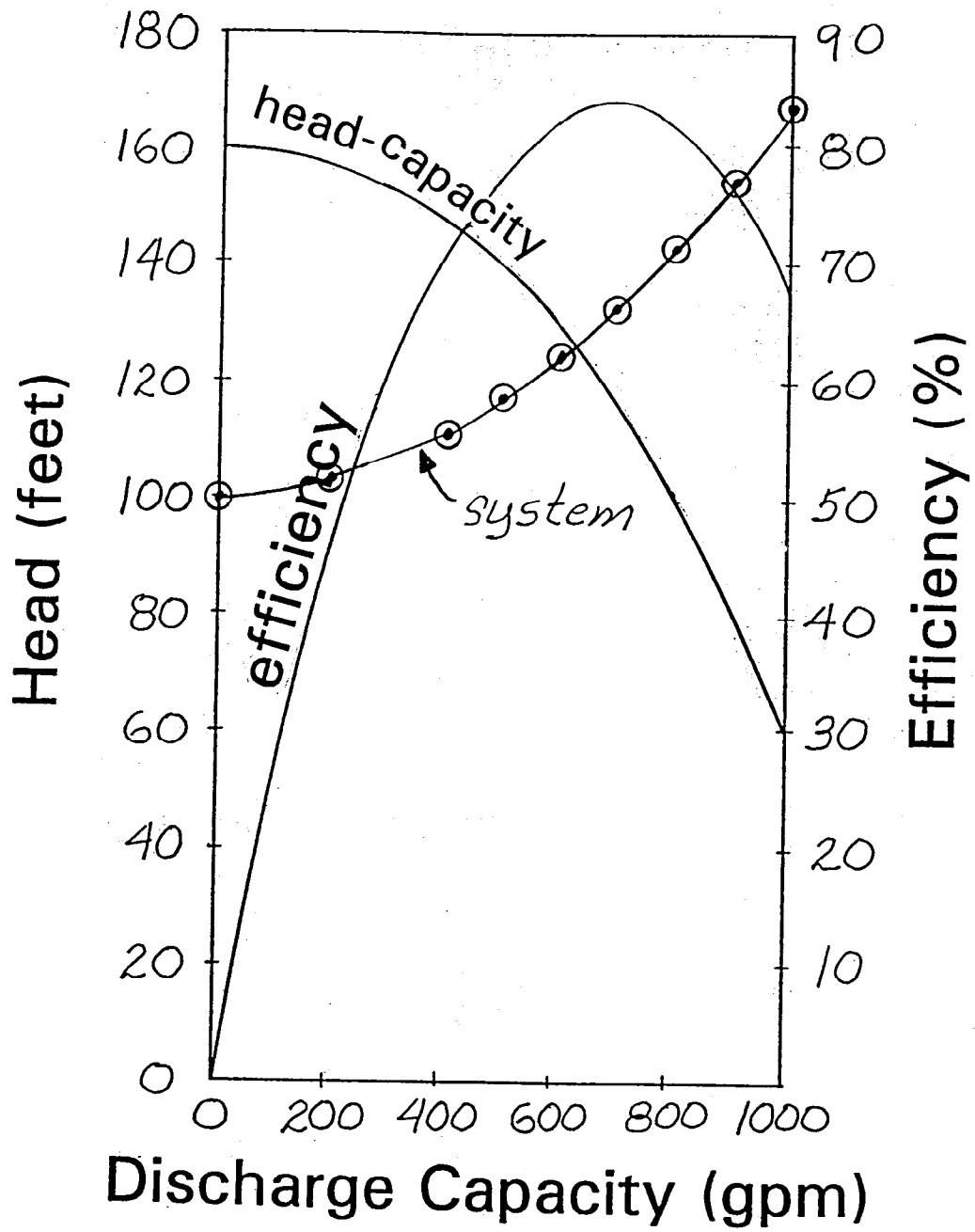
$$h_p = \Delta z + h_L$$

System Head Curve

$$h_p = \Delta z + h_L$$

$$h_p = \Delta z + f(L/D)(Q^2/2gA^2)$$

$$h_p = 100 + 0.020(850/0.5)$$
$$(Q^2/2(32.2)(0.19635)^2)$$

$$h_p = 100 + 13.694 \, Q^2$$

Q (gpm)	Q (cfs)	h_p (ft)
0	0	100.0
200	0.445	102.7
400	0.891	110.9
500	1.114	117.0
600	1.336	124.4
700	1.559	133.3
800	1.782	143.5
900	2.004	155.0
1000	2.227	167.9

$Q = 630$ gpm

$\eta = 83$ %

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Open Channel Flow
Flow Classification

- uniform - flow characteristics (discharge, velocity, depth) are constant along the length of channel

- nonuniform (varied) - flow characteristics vary along the length of channel

  - gradually varied
  - rapidly varied

- steady - flow characteristics are constant over time

- unsteady - flow characteristics change with time

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Uniform Flow

1. The depth, cross-sectional area, velocity, and discharge at every section are constant.

2. The energy line, water surface, and channel bottom are parallel. Their slopes are equal.

normal depth (y_n) - depth at which uniform flow occurs.

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Manning Formula

English Units

$$V = \frac{1.486}{n} R^{2/3} S^{1/2}$$

$$Q = \frac{1.486}{n} AR^{2/3} S^{1/2}$$

Metric Units

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

$$Q = \frac{1}{n} AR^{2/3} S^{1/2}$$

Q = discharge (ft^3/s , m^3/s)

V = Q/A = velocity (ft/s , m/s)

A = cross-sectional area (ft^2, m^2)

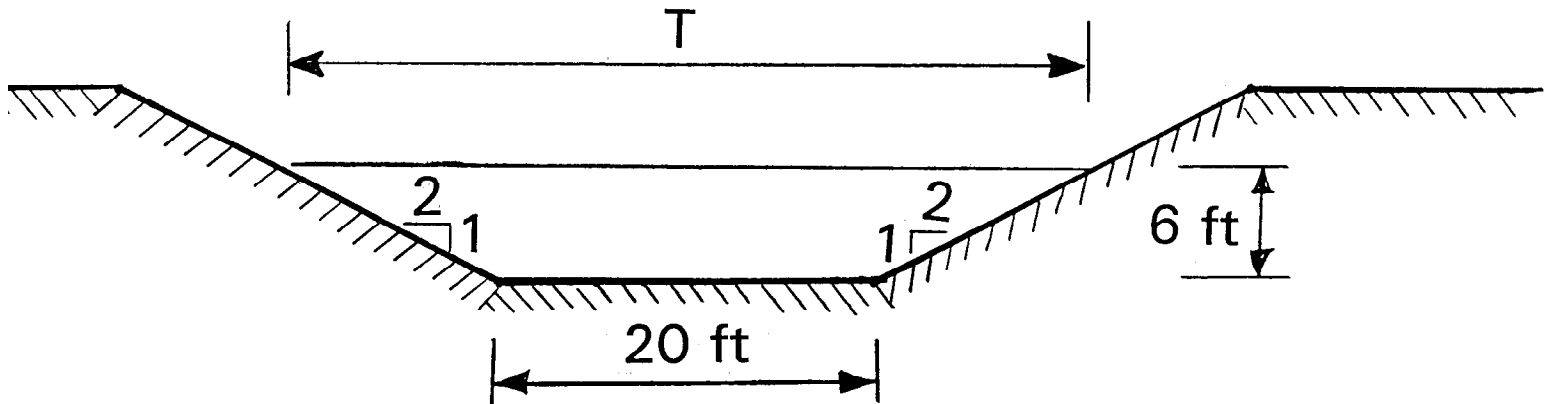R = A/P = hydraulic radius (ft, m)

S = slope

Z = $AR^{2/3}$ = section factor

n = roughness coefficient

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Geometric Elements of Channel Section

example: trapezoidal section

top width $T = 2[\,2(6\ ft)\,] + 20\ ft \ = \ 44\ ft$

flow area

$$A \ = \ \left(\frac{44\,ft \ + \ 20\,ft}{2}\right)(6\,ft) \ = \ 192\ ft^2$$

wetted perimeter
$P \ = \ 20\ ft \ + \ 2\sqrt{(6\ ft)^2 + (12\ ft)^2} \ = \ 46.8\ ft$

hydraulic radius
$R \ = \ A/P \ = \ 192\ ft^2/46.8\ ft \ = \ 4.10\ ft$

hydraulic depth
$D \ = \ A/T \ = \ 192\ ft^2\ /\ 44\ ft \ = \ 4.36\ ft$

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<u>Example</u>: Compute the discharge in a concrete (n=0.015) channel with the previous cross-section and slope of 0.10%.

$$Q = (1.486/n) \, AR^{2/3} \, S^{1/2}$$

$Q = (1.486/0.015)(192ft^2)(4.10ft)^{2/3}(0.001)^{1/2}$

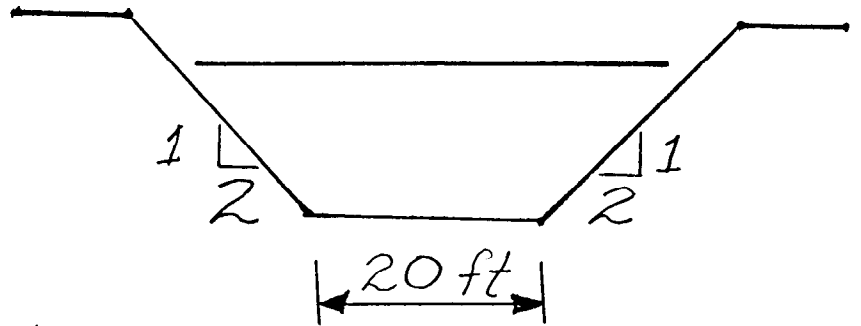$$Q = 1,540 \; ft^3/s$$

Example: Compute the normal depth and velocity.

Q = 400 cfs
S = 0.0016
n = 0.025

$$Q = \frac{1.486}{n} AR^{2/3} S^{1/2}$$

$$AR^{2/3} = \frac{Qn}{1.486 \, S^{1/2}} = \frac{400(0.025)}{1.486(0.0016)^{1/2}}$$

$$AR^{2/3} = 168.2$$

$$A = y\left(\frac{b+T}{2}\right) = y\left(\frac{20+20+2(2y)}{2}\right) = y(20+2y)$$

$$P = b + 2\sqrt{y^2 + (2y)^2} = 20 + 2\sqrt{y^2 + (2y)^2}$$

$$= 20 + 4.472 \, y$$

$$R = A/P = \frac{y(20+2y)}{20+4.472y}$$

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$$AR^{2/3} = 168.2$$

$$Y(20+2Y)\left(\frac{Y(20+2Y)}{20+4.472Y}\right)^{2/3} = 168.2$$

by trial and error

$$Y = Y_n = 3.36 \text{ ft}$$

$$V = \frac{Q}{A} = \frac{Q}{Y(20+2Y)}$$

$$V = \frac{400}{3.36(20+2(3.36))}$$

$$= 4.46 \text{ ft/s}$$