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ELEN TAMU

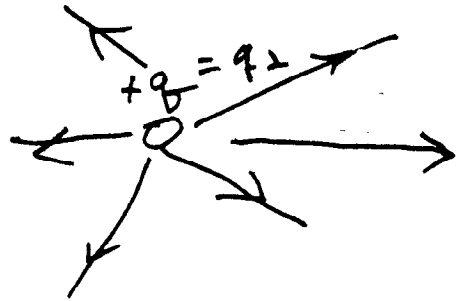
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# Electric Circuits

1. Fundamentals
2. Resistive Circuits with DC
3. Circuit Solution Techniques
4. Capacitors and Inductors
5. First Order Transients
6. AC Signals
7. AC Circuits
8. Op Amps

# 1. Fundamentals

Charge  $Q$  or  $q(t)$



$\leftarrow 0 - q = q_1$   
 $\hat{a}_{12}$   
 unit vector

Coulomb's Force  $\vec{F}$

$$\vec{F}_{12} = \hat{a}_{12} \frac{q_1 q_2}{r_{12}^2} k$$

$$= q_1 \left( \hat{a}_{12} \frac{q_2}{r_{12}^2} k \right)$$

$$= q_1 \vec{E}_2$$

$\vec{F}_{12}$  attractive if  $q_1 q_2 < 0$

$\vec{F}_{12}$  repulsive if  $q_1 q_2 > 0$

# Electric Fields $\vec{E}$ , $\vec{D}$

$\vec{D}$  - displacement

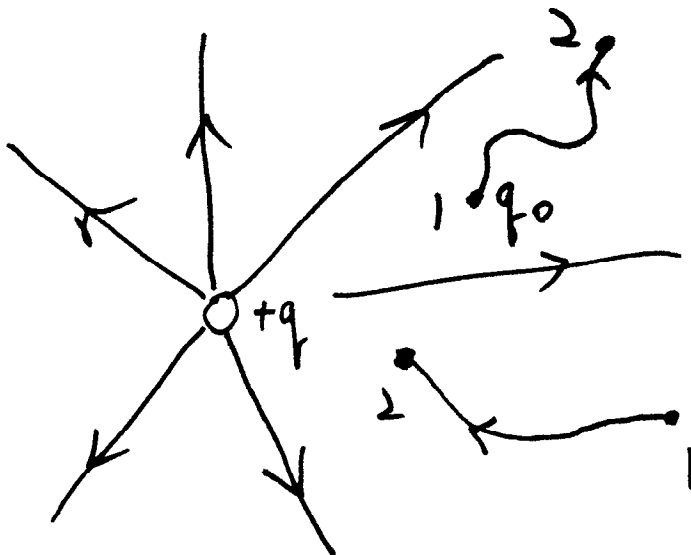
- electric flux dens.

$\vec{E}$  - electric field

$$\vec{D} = \epsilon \vec{E}$$

## Voltage $V$ , $v(t)$ , $\mathcal{E}$ or $e$

↳ potential difference



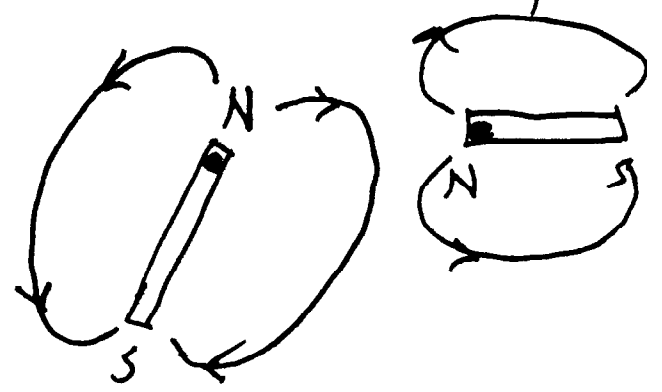
$$\Delta V = [\text{Volts}]$$

# Magnetic Fields $\vec{H}, \vec{B}$

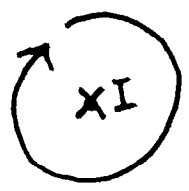
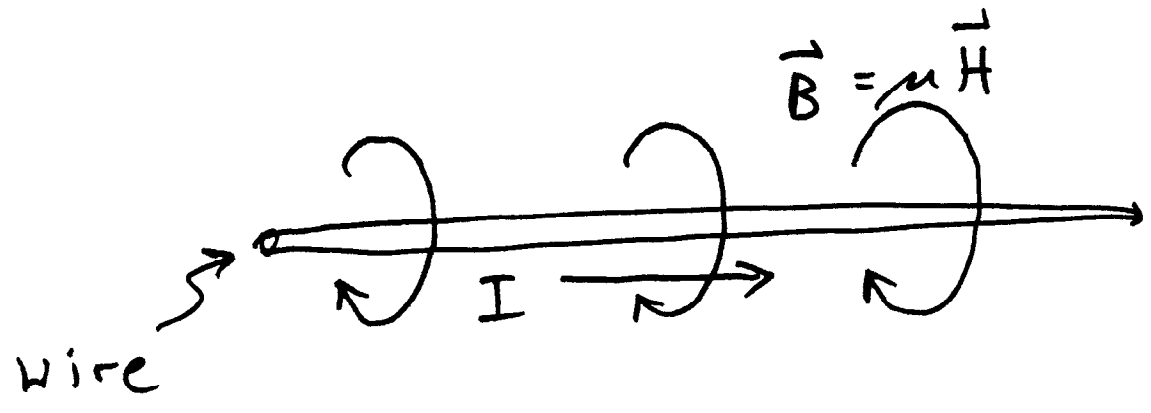
$\vec{B}$  = mag. flux density

$\vec{H}$  = mag. field intensity

$$\vec{B} = \mu \vec{H}$$



# Current $I, i(t)$



I going into page

right hand rule

# Induced Voltage $v$ or $e$

## Faraday's Law

$$v = - \frac{d}{dt} \Phi$$

$\uparrow$  mag flux

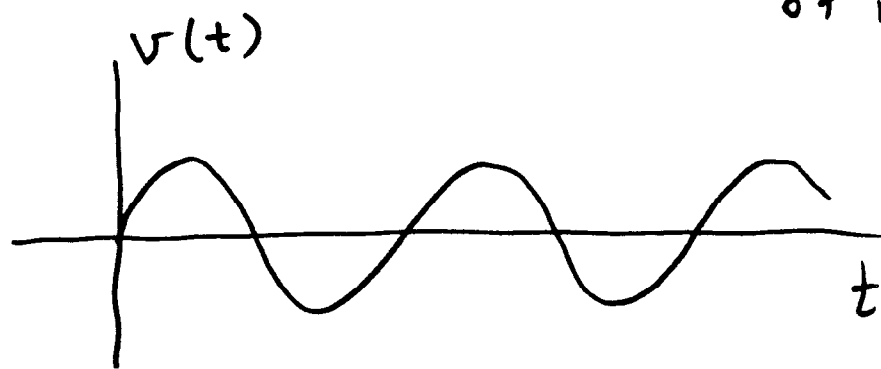
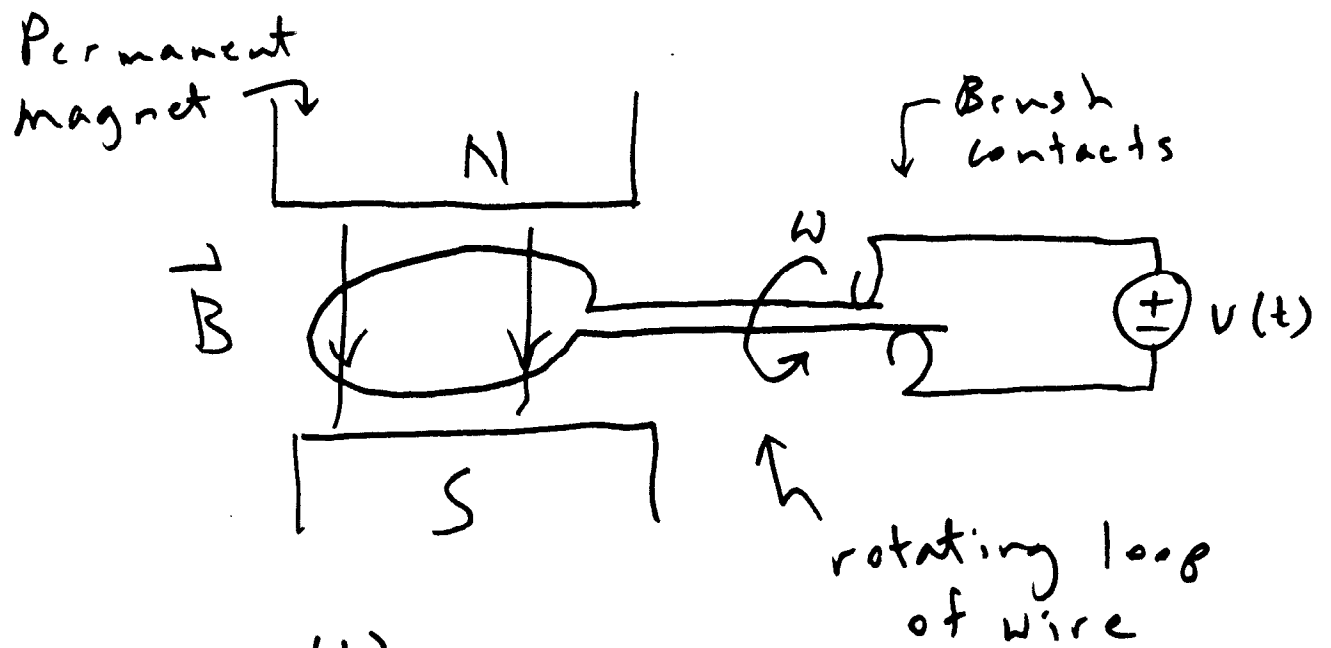
$$\Phi = \oint \vec{B} \cdot d\vec{a}$$

$$= \int B A \cos(90^\circ) da$$

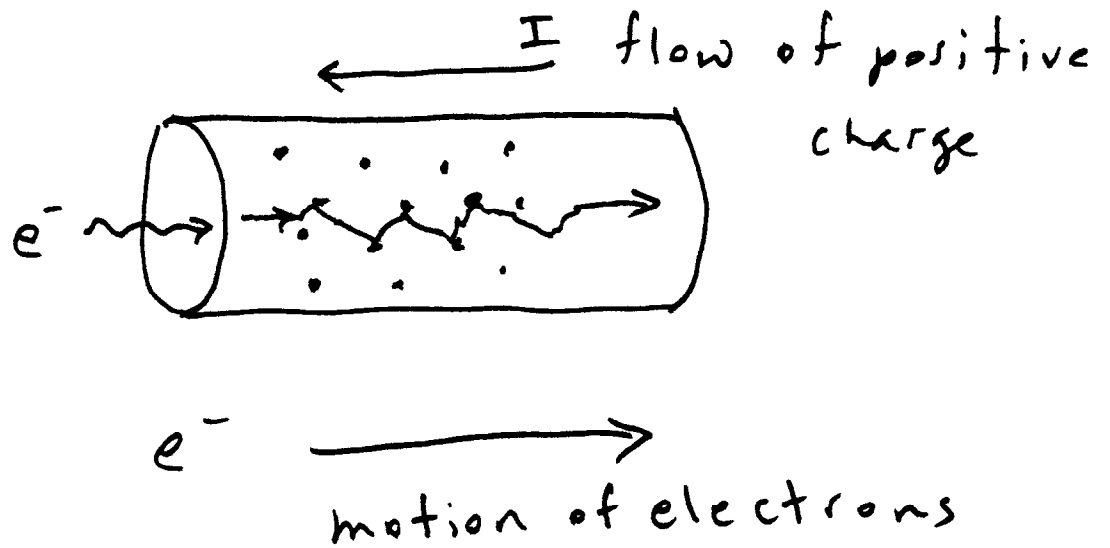
$$= \underline{B \cdot A} \quad \text{special case}$$

$$v(t) = - \frac{d}{dt} B \cdot A$$

1-4.1

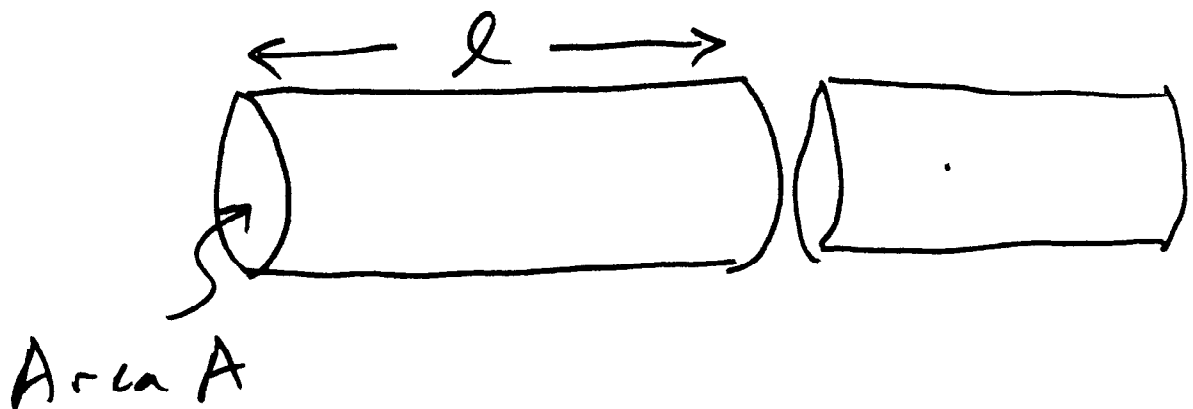


# Resistivity $\rho$



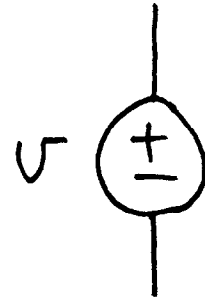
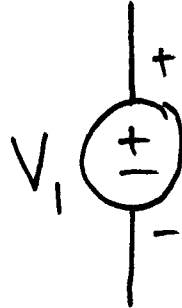
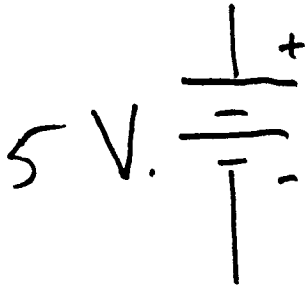
# Resistance $R$

$$R = \rho \frac{l}{A} = [\text{ohms}] = [r]$$





## Voltage Sources

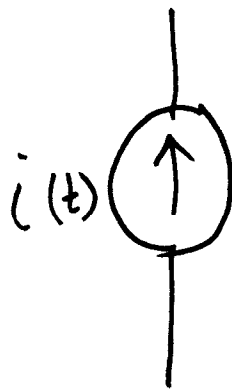
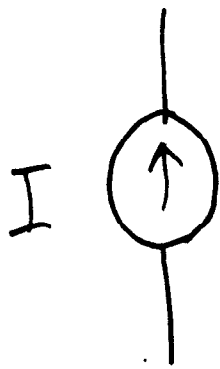


$$V_1 = 2 \text{ V.}$$

$$v(t) = 2 \cos(3t)$$

[Volts]

## Current Sources



$$i(t) = 4 \cos(800\pi t)$$

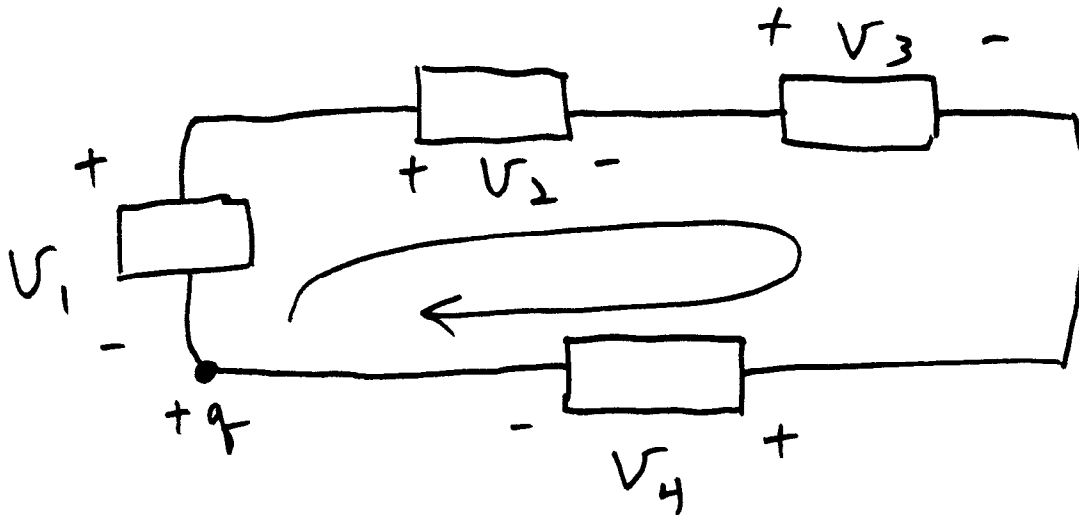
$$I = 3 \text{ A.}$$

[Amps]

# Kirchhoff's Laws

1-7

$$\text{KVL: } \sum_{m=1}^N V_m = 0$$

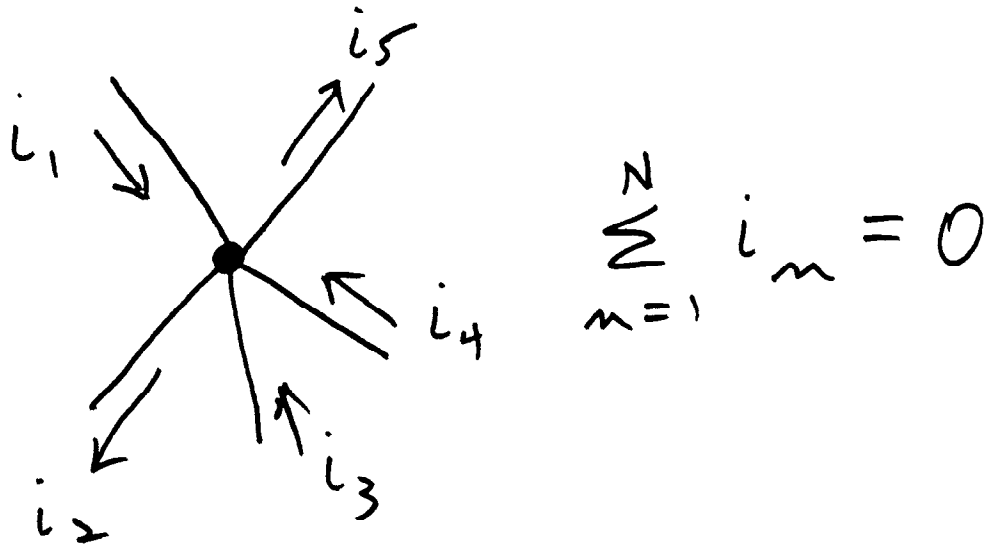


$$-V_1 + V_2 + V_3 + V_4 = 0$$

$$+V_1 - V_2 - V_3 - V_4 = 0$$

energy conservation for circuits

KCL - charge is conserved



$$+i_1 - i_2 + i_3 + i_4 - i_5 = 0$$

$$i_2 = i_1 + i_3 + i_4 - i_5$$

$$i_1 + i_3 + i_4 = i_2 + i_5$$

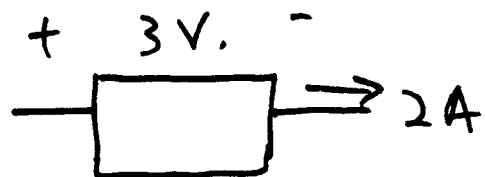
# Electric Power, Energy

1-8

$$\text{Power} = [\text{Watts}]$$

$$\text{Energy} = [\text{Joules}]$$

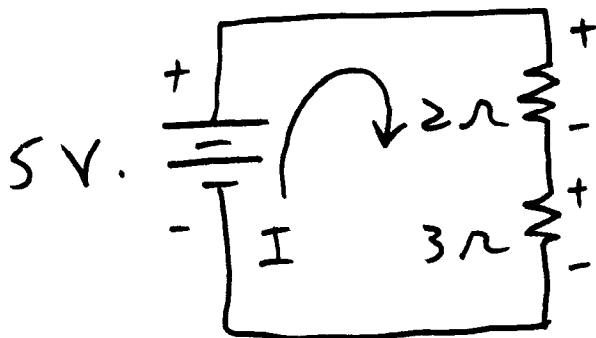
$$P = VI$$



$$= 3 \cdot 2 = 6 \text{ W.}$$

## Power Balance

$$P_{\text{generated}} = P_{\text{dissipated}}$$



Ohm's  
Law

$$V = RI$$

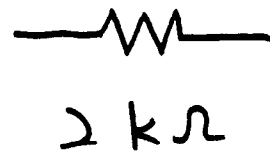
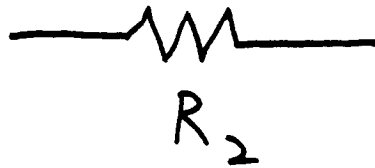
5 Watts gen.

## 2. Resistive Circuits

Ohm's Law  $V = R I$

$$= \left( \rho \frac{\ell}{A} \right) I$$

Circuit Symbol



k-kilo M-mega

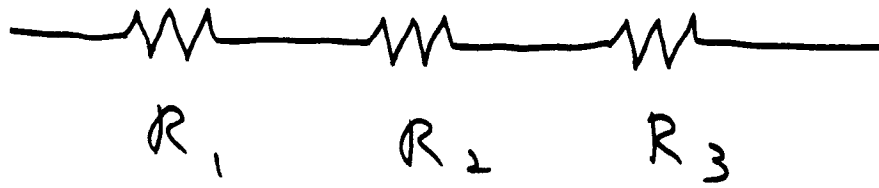
Power in resistors

$$P = VI$$

$$= V \left( \frac{V}{R} \right) = \frac{V^2}{R}$$

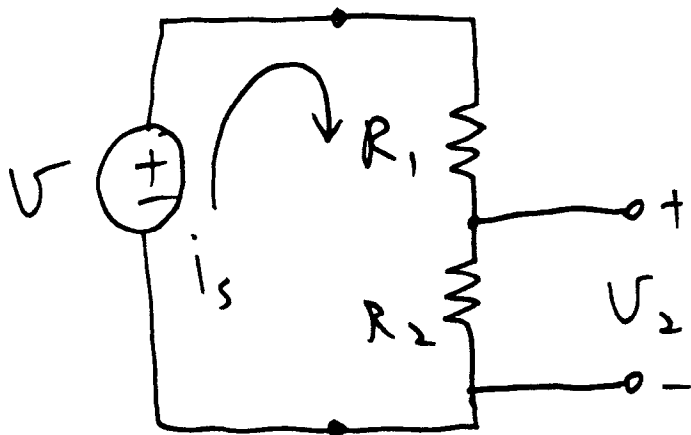
$$= (RI)I = RI^2$$

## Resistors in Series



$$R_{eq} = R_1 + R_2 + R_3$$

## Voltage Divider



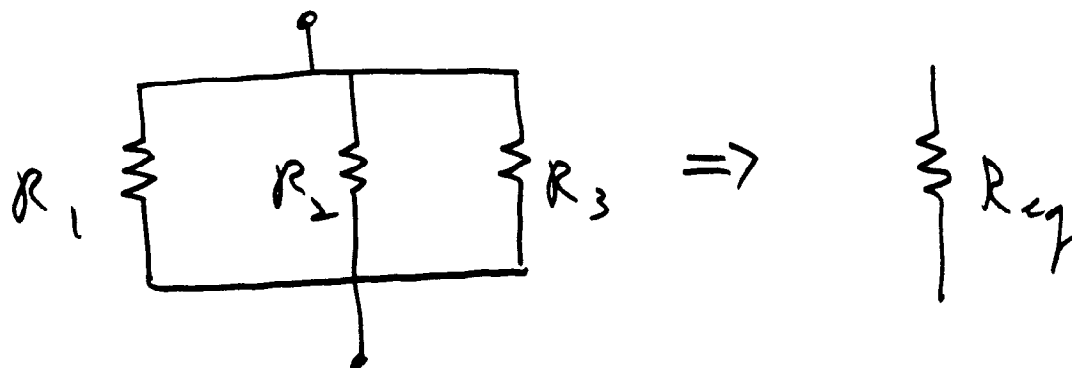
$$i_s = \frac{V}{R_{eq}}$$

$$= \frac{V}{R_1 + R_2}$$

$$V_2 = R_2 i_s$$

$$= V \left( \frac{R_2}{R_1 + R_2} \right)$$

## Resistors in Parallel

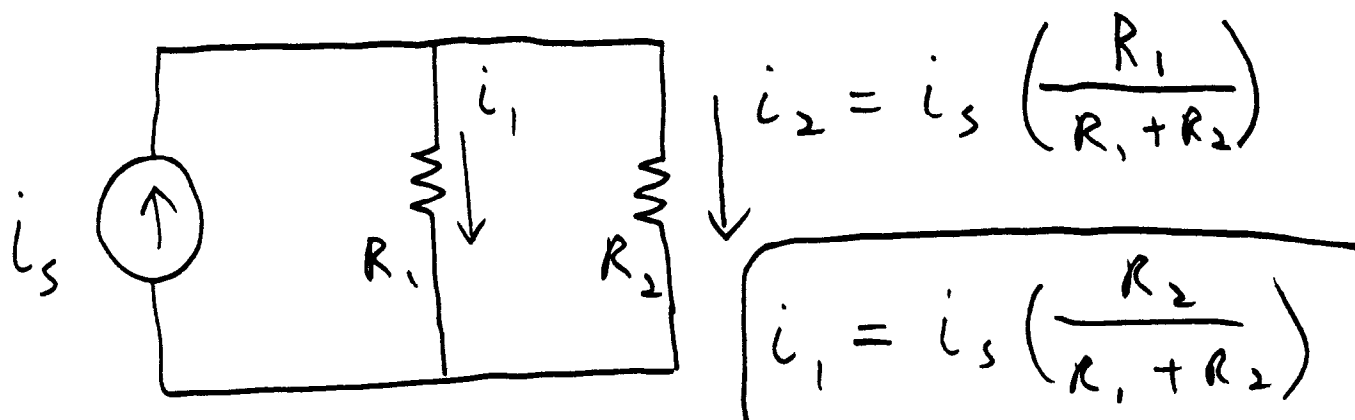


$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Two Resistors (special case)

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2} \quad \frac{[\Omega]^2}{[\Omega]}$$

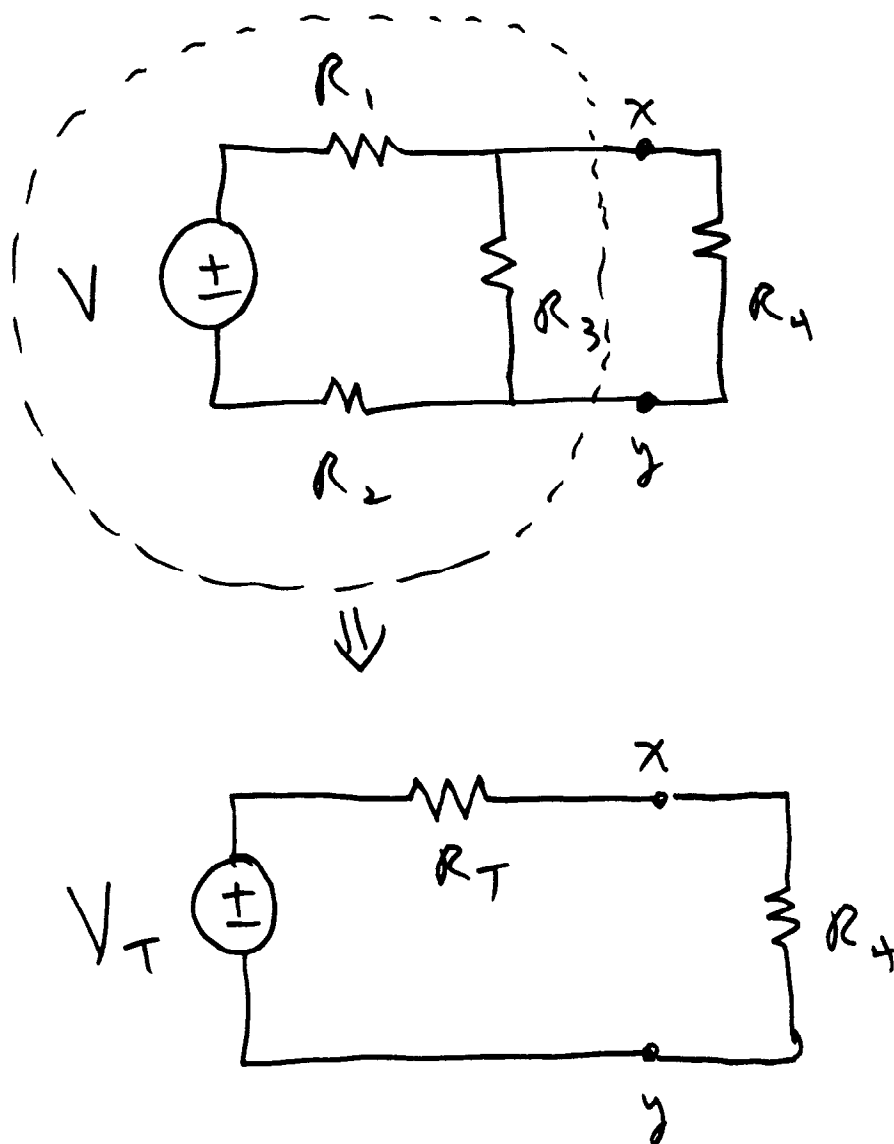
## Current Divider



# Thevenin Equivalent

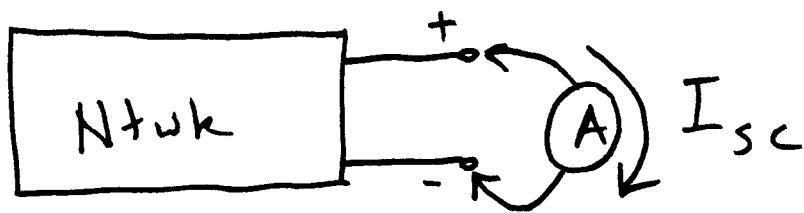
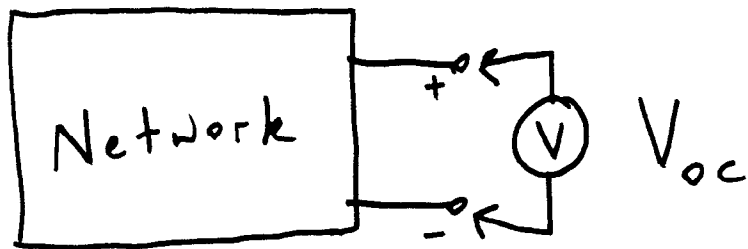
$V_{oc}$  ,  $I_{sc}$  ,  $V_T$  ,  $R_T$

Network - interconnection of  
Sources + others



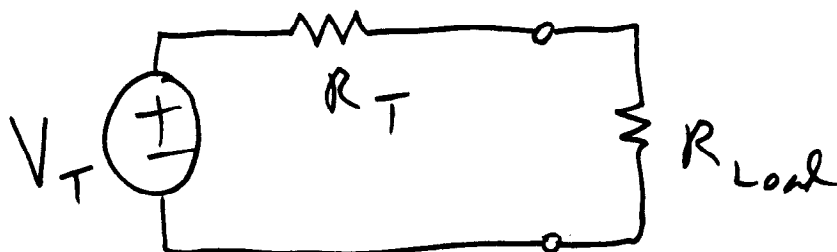


$V_{oc}$  - open circuit voltage



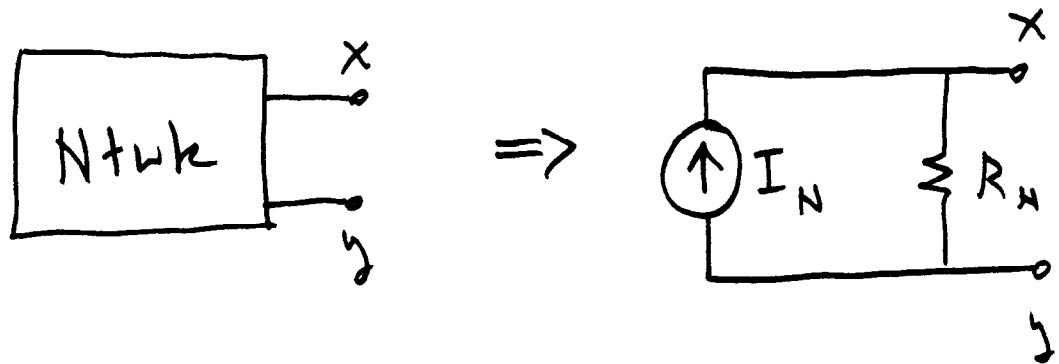
Ther.  
Equiv.

$$\begin{cases} V_T = V_{oc} \\ R_T = \frac{V_{oc}}{I_{sc}} \end{cases}$$



## Norton Equivalent

$$V_{oc}, I_{sc}, I_N, R_N$$

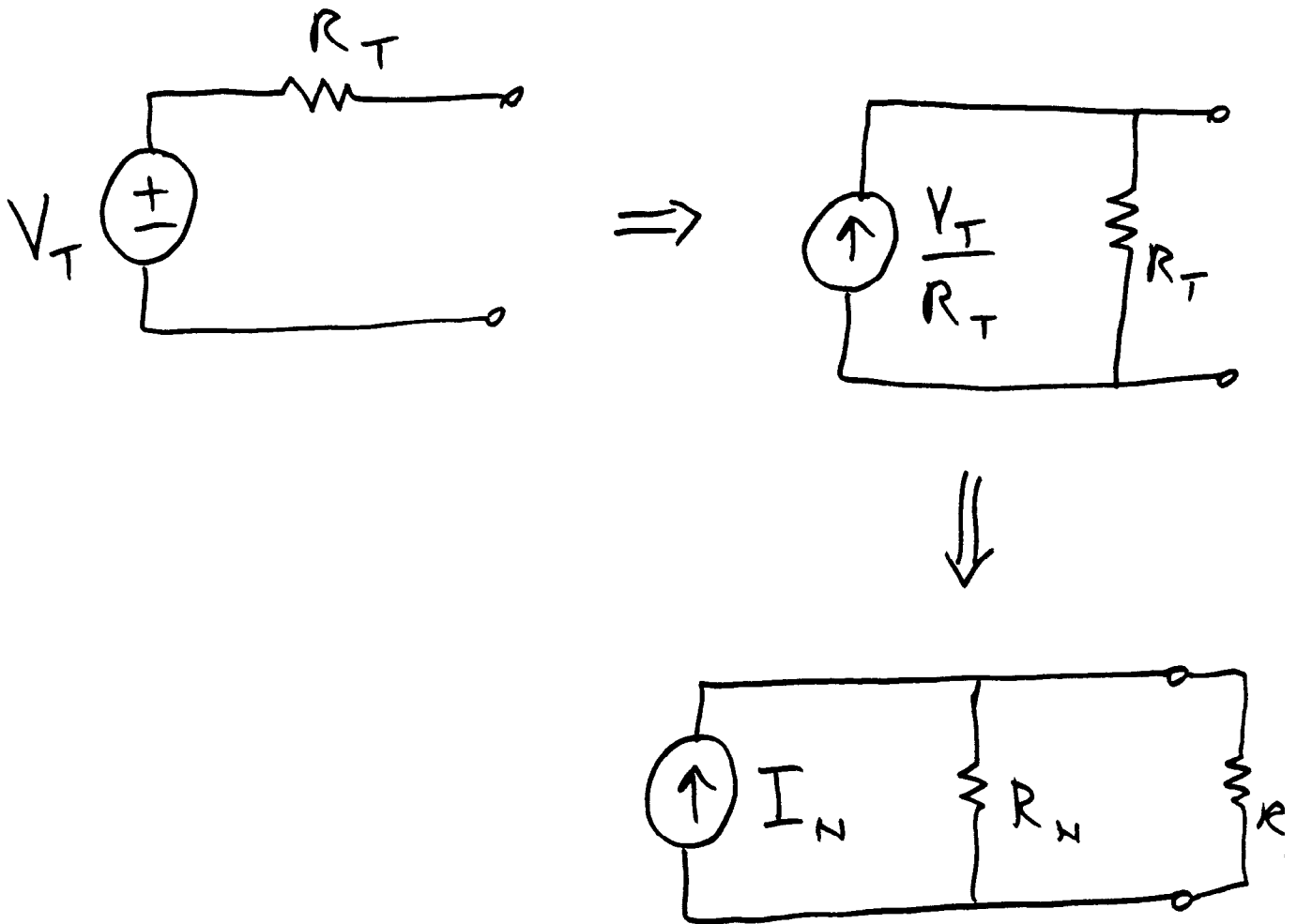


$$\text{Norton Equiv.} \begin{cases} I_N = I_{sc} \\ R_N = \frac{V_{oc}}{I_{sc}} \quad (= R_T) \end{cases}$$

$$V_{oc} = R_N I_{sc} = R_N I_N$$

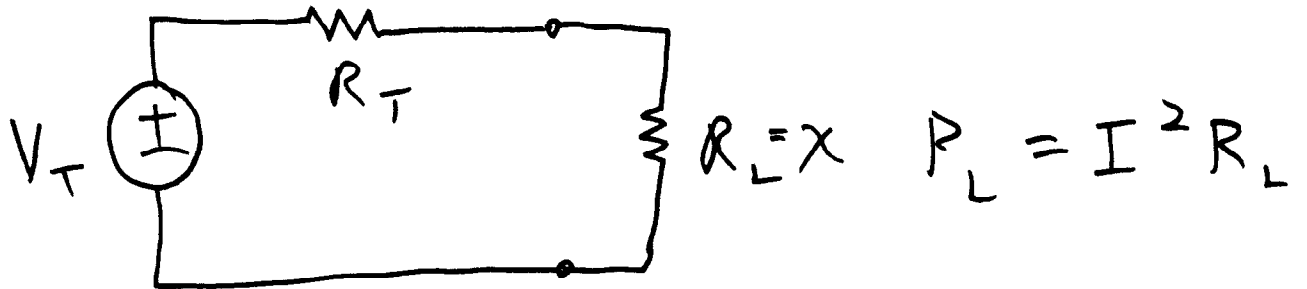
$$V_T = R_N I_N$$

## Source Transformation



i.e. Thevenin  $\leftrightarrow$  Norton

## Maximum Power Transfer



What  $R_L$  gives maximum  $P_L$ ?

$$I = \frac{V_T}{R_{eq}} = \frac{V_T}{R_T + x}$$

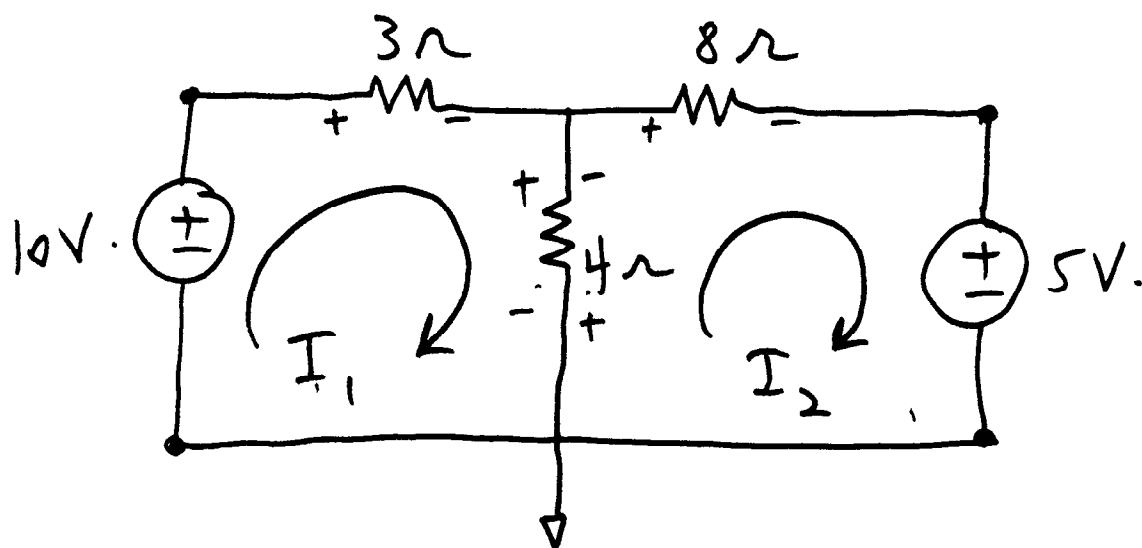
$$P_L = \left( \frac{V_T}{R_T + x} \right)^2 x$$

$$= V_T^2 \frac{x}{(R_T + x)^2}$$

$$\frac{dP_L}{dx} = 0 \longrightarrow \underline{x = R_T}$$

### 3. Circuit Solution Techniques

#### Loop Currents



KVLs

$$\text{loop 1: } -10 + 3I_1 + 4(I_1 - I_2) = 0$$

$$\text{loop 2: } +5 + 4(I_2 - I_1) + 8I_2 = 0$$

$$7I_1 - 4I_2 = 10$$

$$-4I_1 + 12I_2 = -5$$

Solve for  $I_1, I_2$

3-1.1

$$7I_1 - 4I_2 = 10$$

$$-4I_1 + 12I_2 = -5$$

Ans:  $\underline{I_1 = 1.47 \text{ [A]}}$ ,  $I_2 = 0.0735$   
 $\underline{I_2 = 73.5 \text{ [mA]}}$

Check:

$$-10 \text{ V.} + (3\Omega)(I_1) + (8\Omega)(I_2) + 5 = ?$$

KVL around  $\nearrow$   
 outside loop

$$0 \approx -0.002$$

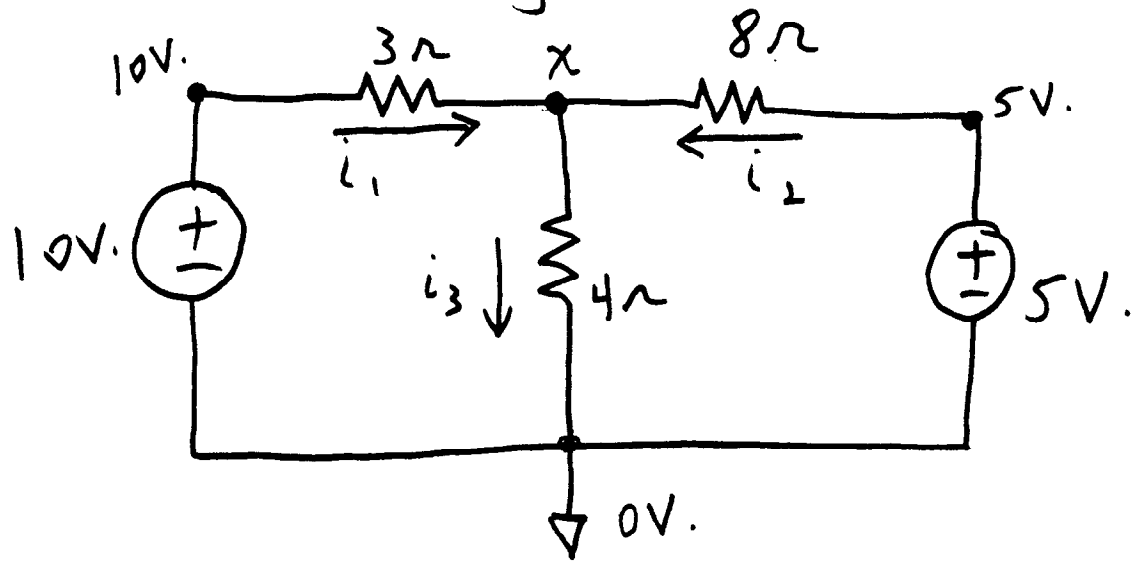
Also loop 1 KVL check

$$-10 \text{ V.} + (3\Omega)I_1 + (4\Omega)(I_1 - I_2) \stackrel{?}{=} 0$$

$$-10 + 3(1.47) + 4(1.47 - 0.0735) \stackrel{?}{=} 0$$

$$-0.004 \approx 0$$

## Node Voltages



Node voltage at  $x$ :  $V_x$

KCL at  $x$ :  $i_1 + i_2 = i_3$

$$\left(\frac{10 - V_x}{3}\right) + \left(\frac{5 - V_x}{8}\right) = \left(\frac{V_x - 0}{4}\right)$$

$$8(10 - V_x) + 3(5 - V_x) = 6V_x$$

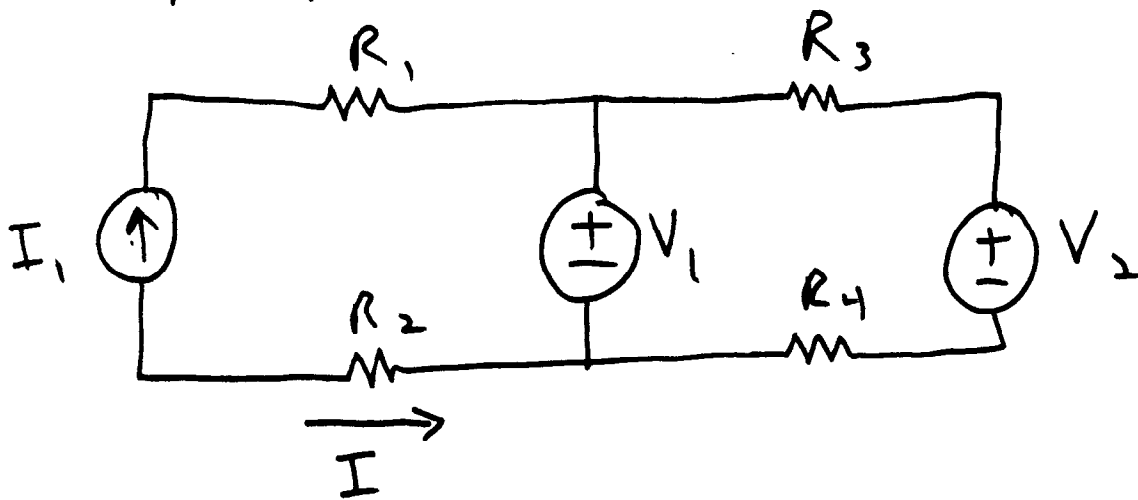
$$V_x = 5.59 \text{ V.}$$

Compare

From loop currents:

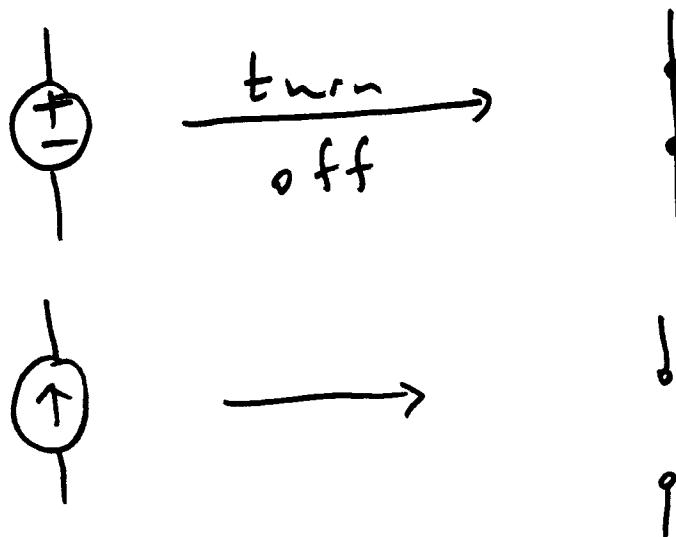
$$4(I_1 - I_2) = 4(1.47 - 0.0735) \cong 5.59$$

## Superposition



$$I = I_{cs} + I_{vs1} + I_{vs2}$$

↳ "turn off" the other two sources



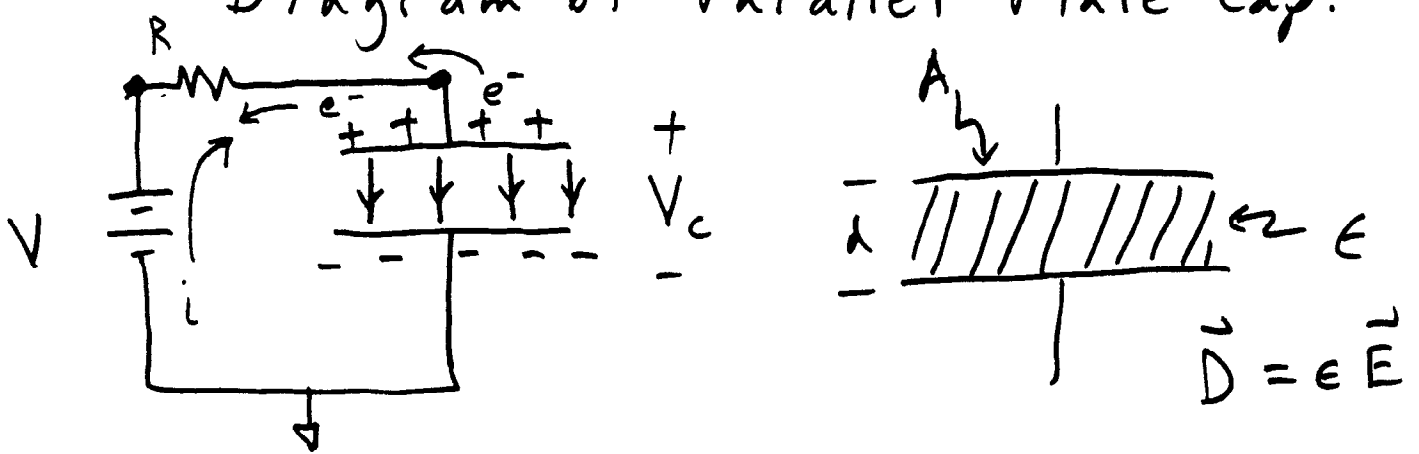


## 4. Capacitors + Inductors

Capacitor Symbol  $\frac{\perp}{\perp} C \frac{\perp}{\perp}$

$C = [\text{Farad}] \quad \text{mF} \quad \mu\text{F} \quad \text{nF}$

Diagram of Parallel Plate Cap.



Basic Theory  $Q = CV$

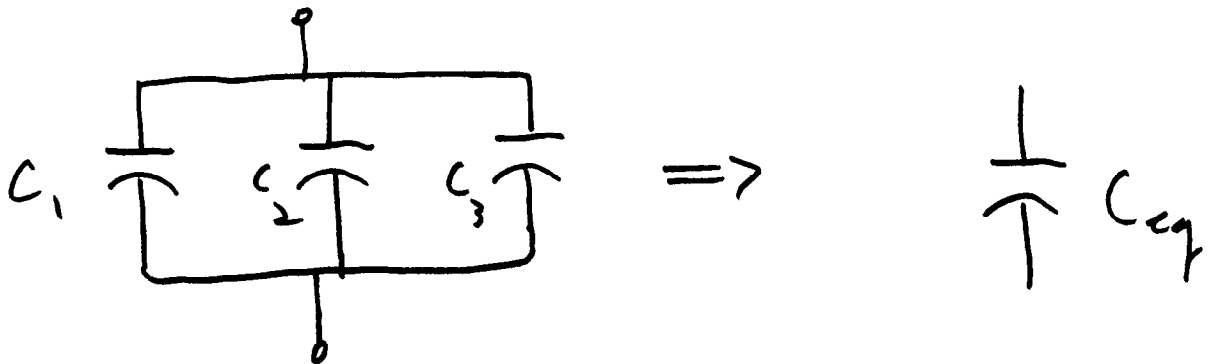
$$C = \epsilon \frac{A}{d}$$

$$q(t) = C v(t)$$

Math model:  $i(t) = C \frac{d}{dt} v(t)$

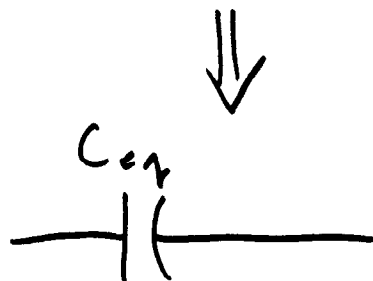
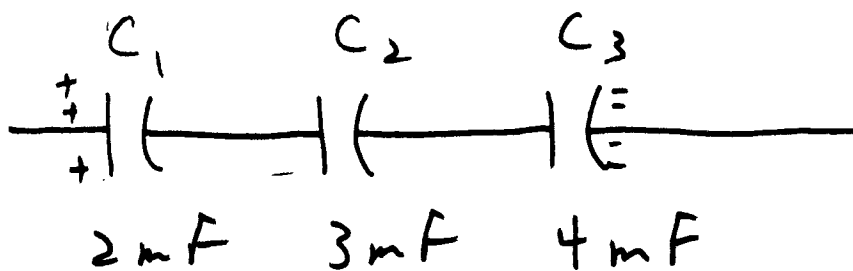
Recall:  $\frac{dq(t)}{dt} \equiv i(t) = [A.] = \left[ \frac{\text{Coul}}{s} \right]$

## Capacitors in Parallel



$$C_{eq} = C_1 + C_2 + C_3$$

## Capacitors in Series



$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

# Energy Storage in Capacitors 4-3

$$w = \int p dt \quad p = \frac{dw}{dt}$$

$$= \int v i dt$$

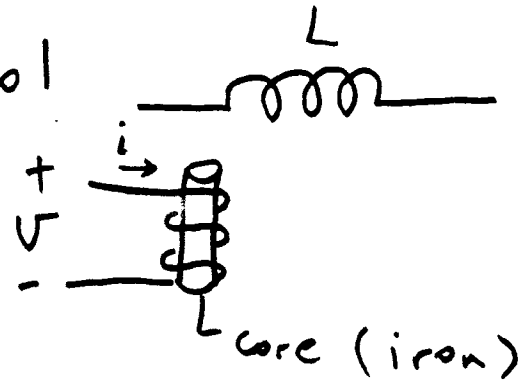
$$w_c = \int v \left( C \frac{dv}{dt} \right) dt$$

$$= C \int v dv$$

$$= C \frac{v^2}{2}$$

$$w_c(t) = \frac{1}{2} C [v_c(t)]^2$$

Inductor Symbol



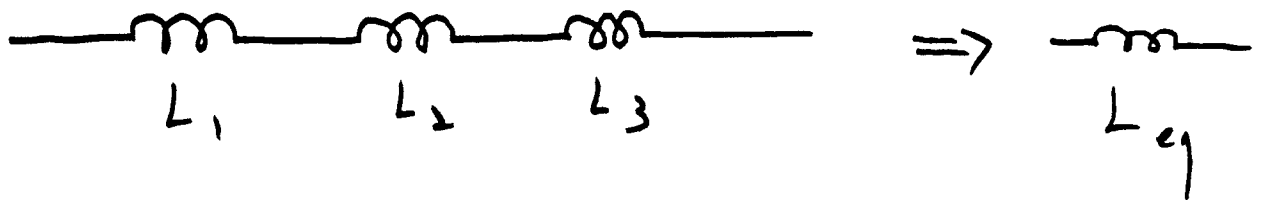
Basic theory

Math model:

$$v(t) = L \frac{d}{dt} i(t)$$

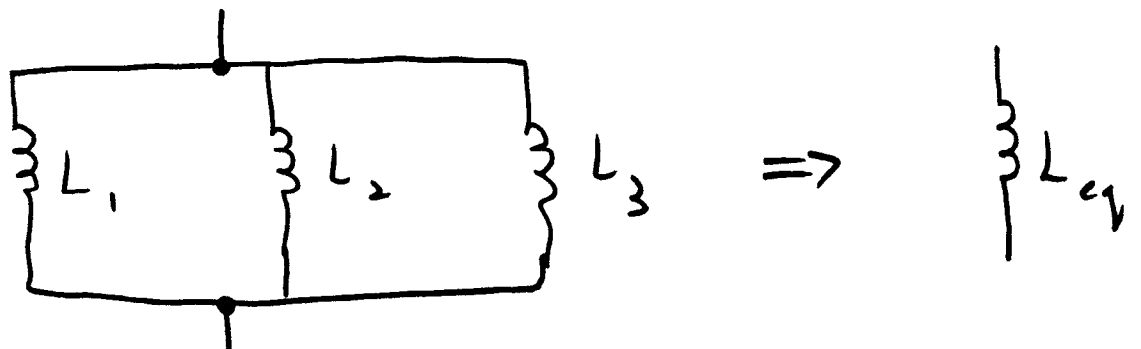
$$L = [\text{Henry}]$$

Inductors in series



$$L_{eq} = L_1 + L_2 + L_3$$

Inductors in parallel



$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

## Energy Storage in Inductors

$$W = \int p \, dt$$

$$= \int v i \, dt$$

$$W_L = \int L \frac{di}{dt} i \, dt$$

$$= L \int i \, di$$

$$W_L(t) = \frac{1}{2} L [i(t)]^2 = [J]$$

# 5. First Order Transients

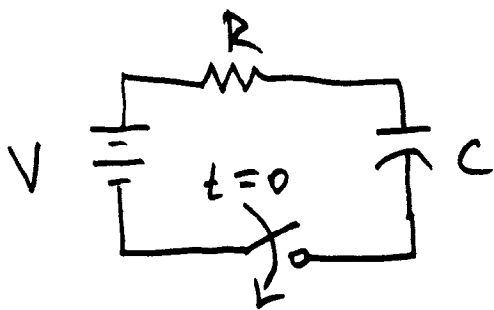
5-1

"transient" vs. "steady state"

↑ changing

↑

constant



Capacitor in steady state

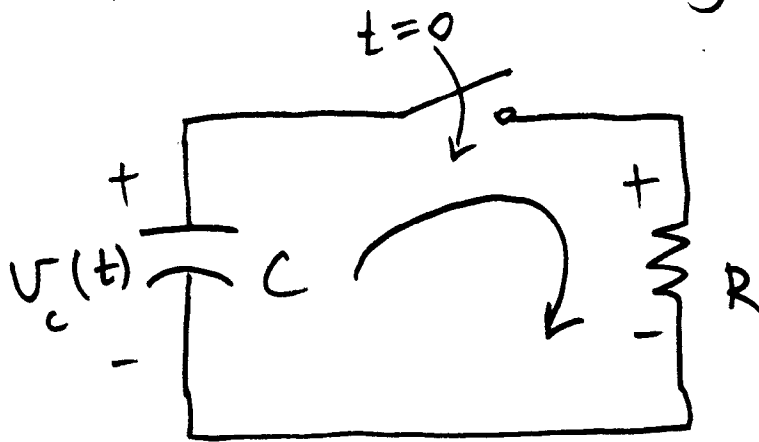
$$i = C \frac{dv}{dt} = C \frac{d}{dt} (\text{constant})$$

$$\left. \begin{array}{l} i = 0 \\ v \neq 0 \end{array} \right\} \text{like an open circuit}$$

Inductor in steady state

$$v = L \frac{di}{dt} \xrightarrow{\text{s.s.}} \underbrace{v = 0, i \neq 0}_{\text{like a short circuit}}$$

# Capacitor Discharge



$v_c(t)$  = voltage across cap

$v_c(0) = V$  , initially charged

Passive Sign Convention

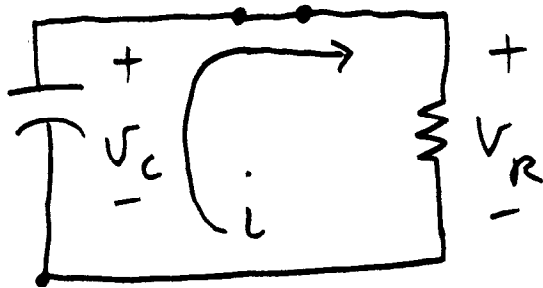
$$i = +C \frac{dv}{dt} \quad \text{iff} \quad \begin{array}{c} + \\ v_c \\ - \end{array} \begin{array}{c} \uparrow \\ \downarrow i_c \end{array}$$

$$\text{If } \begin{array}{c} + \\ v_c \\ - \end{array} \begin{array}{c} \uparrow \\ \downarrow i_c \end{array} \quad \text{then} \quad i = -C \frac{dv}{dt}$$

"active"

↗ delivering energy

5-2.1

For  $t > 0$ 

$$V_c(0) = V$$

$$\text{KVL: } -V_c + V_R = 0$$

$$i = -C \frac{dV}{dt} \rightarrow \underline{V = -\frac{1}{C} \int i dt}$$

passive  $\uparrow$   
sign convention

$$= \frac{-\int i dt}{C}$$

$$= -\frac{q}{C}$$

$$q = -C V$$

$$- \left( -\frac{1}{C} \int i dt \right) + R i = 0$$

$$\frac{1}{C} i + R \frac{di}{dt} = 0$$

$$\frac{di}{dt} + \frac{1}{RC} i = 0$$



5-2.2

$$\frac{di}{dt} + \frac{1}{RC} i = 0$$

solve by assuming  
an exponential sol'n

$$i = A e^{st}$$

$$i(t) = \frac{V}{R} e^{-t/RC}$$

$$\frac{di}{dt} = A s e^{st} = s i$$

$$s i + \frac{1}{RC} i = 0$$

$$s = -\frac{1}{RC}$$

$$i = A e^{-t/RC}$$

$$V(0) = R i(0) = V$$

$$R(A e^0) = V$$

$$A = \frac{V}{R}$$

$$\rightarrow i(t) = \frac{V}{R} e^{-t/R}$$

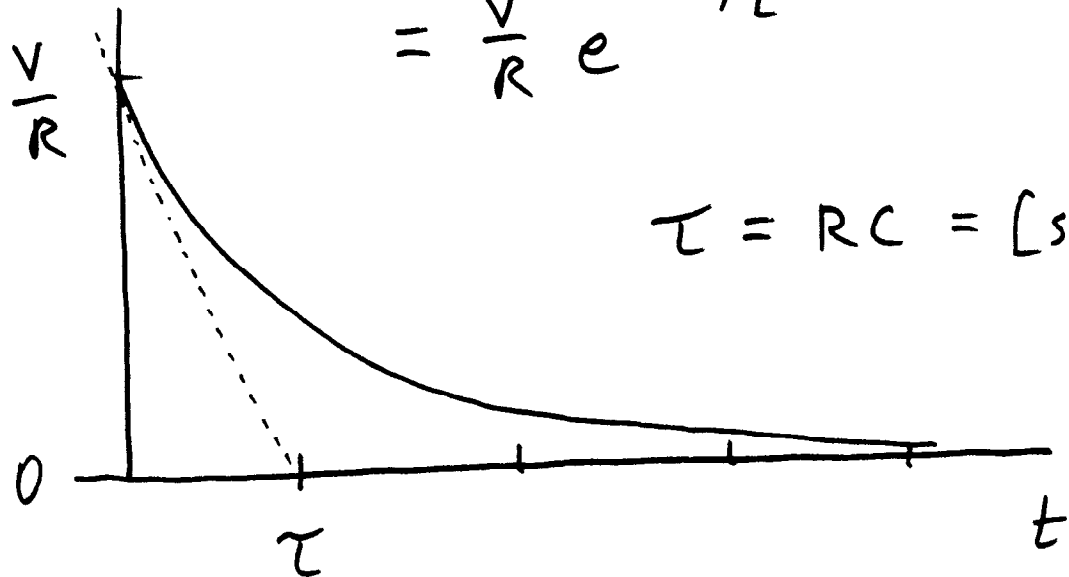
calculation

result

5-2.3

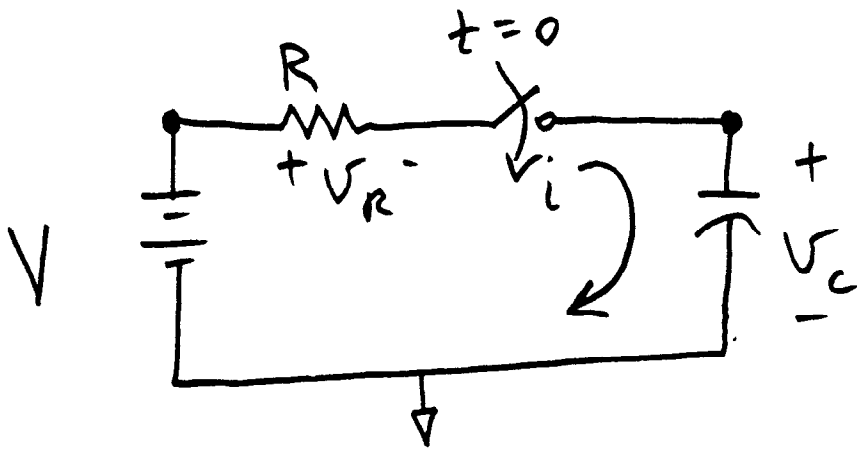
$$i = \frac{V}{R} e^{-t/RC}$$

$$= \frac{V}{R} e^{-t/\tau}$$



Transient period ends  
after  $5\tau$ .

## Capacitor Charging



$$v_C(0) = 0 \text{ [V.]}$$

$$v_C(\infty) = V$$

$$\text{KVL: } -V + v_R + v_C = 0$$

$$-V + Ri + \frac{1}{C} \int i dt = 0$$

$$R \frac{di}{dt} + \frac{1}{C} i = 0$$

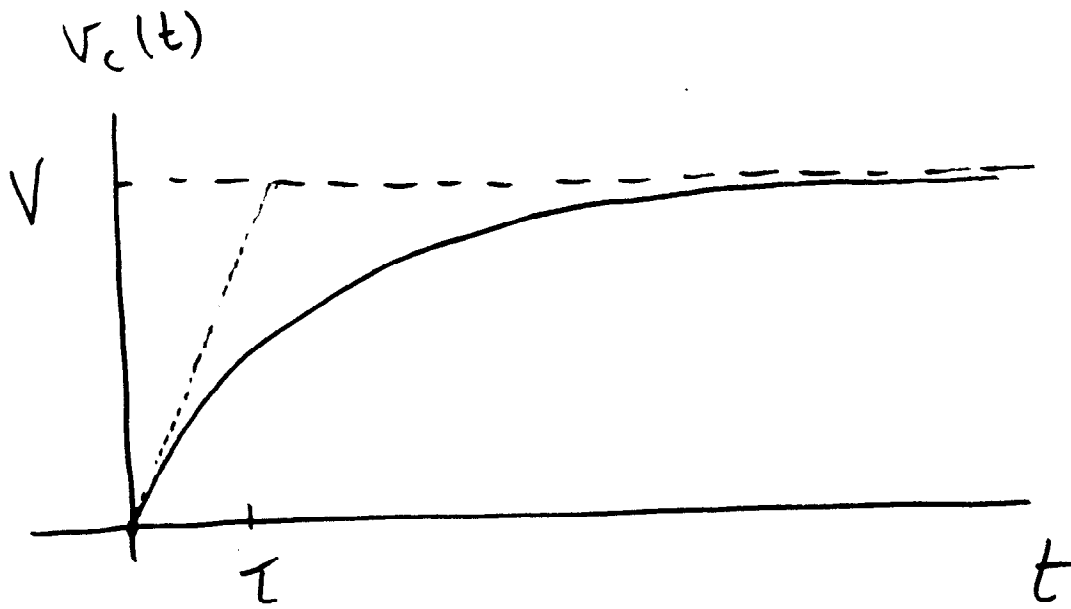
$$\vdots$$

$$i = I_0 e^{-t/RC}$$

$$v_C(t) = V (1 - e^{-t/RC})$$

$$v_C(0) = 0, \quad v_C(\infty) = V$$

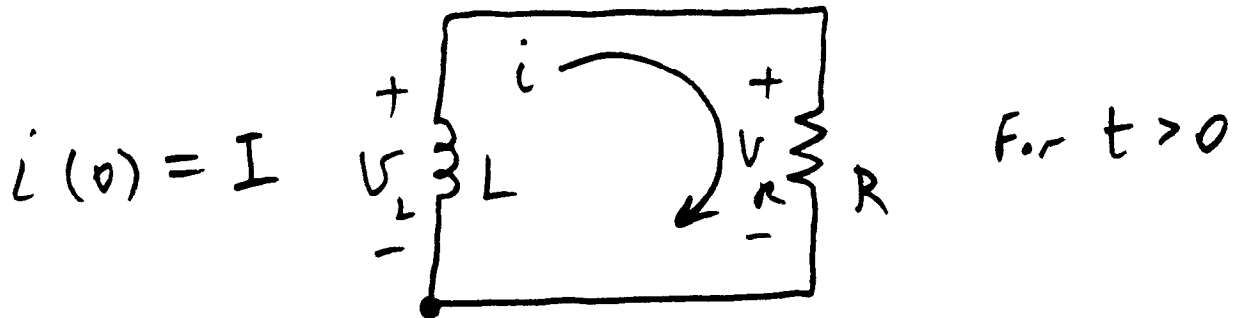
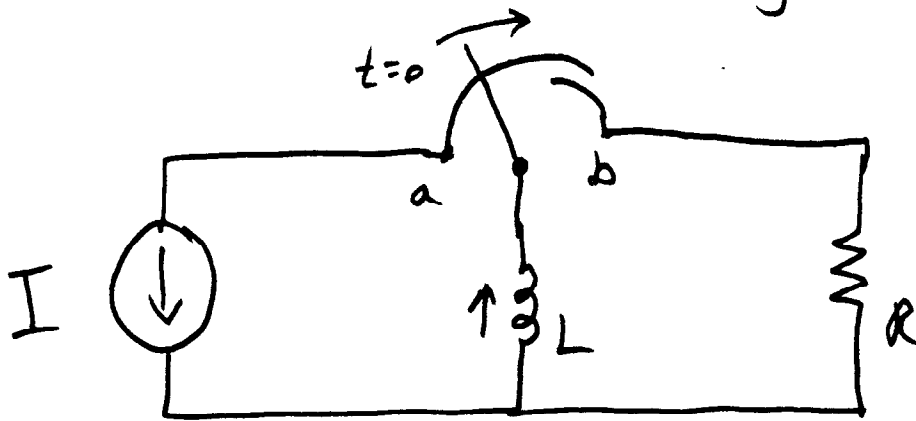
5-3.1



$$L \tau = RC$$

# Inductor "De-Energizing"

5-4



$$V_L = -L \frac{di}{dt} \quad V_R = Ri$$

$$-V_L + V_R = 0$$

$$-(-L \frac{di}{dt}) + Ri = 0$$

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

passive  
sign  
convention

5-4.1

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

$$i = A e^{st}$$

$$s i + \frac{R}{L} i = 0$$

$$s = -\frac{R}{L}$$

$$i(t) = I e^{-\frac{R}{L} t}$$



$$= I e^{-t/\tau}$$

$$\tau = \frac{L}{R} = [s]$$

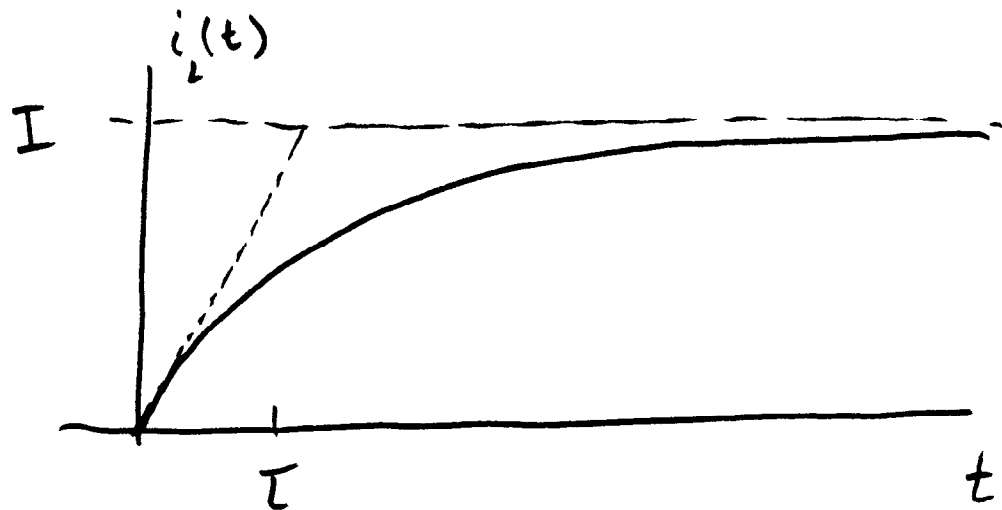
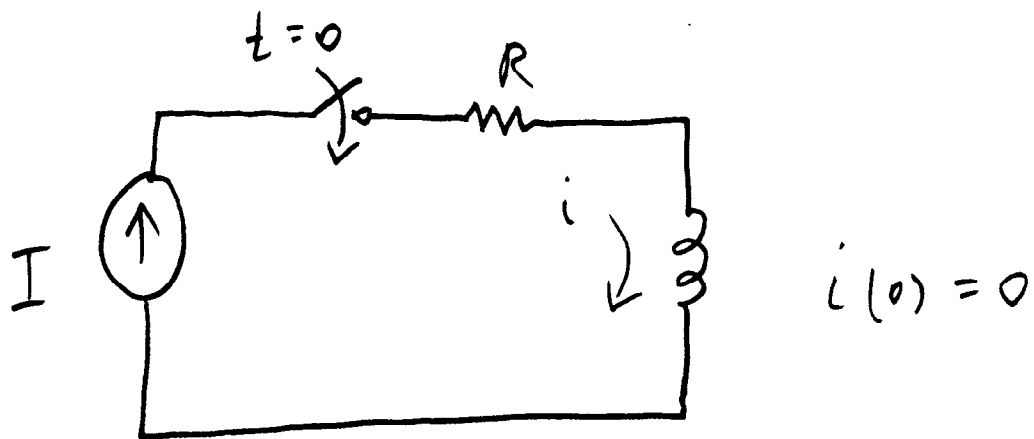
Units

$$V = L \frac{di}{dt} \rightarrow L = \frac{V}{di/dt} = \left[ \frac{V}{A/s} \right]$$

$$[F] = \left[ \frac{As}{V} \right]$$

$$[H] = \left[ \frac{Vs}{A} \right] \checkmark$$

## Inductor Energizing

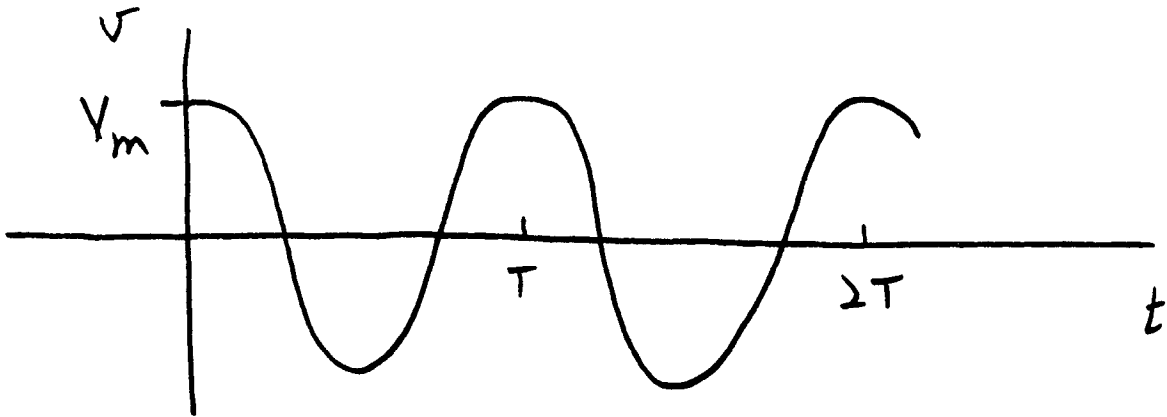


$$i(t) = I(1 - e^{-t/\tau})$$

$$\tau = L/R$$

## 6. AC Signals

Waveform + terms



$V_m$  - magn. or amplitude [V]

$T$  - period [s]

$f$  - frequency [Hz]  $f = \frac{1}{T}$

$$\omega = 2\pi f = \frac{2\pi}{T} = \left[ \frac{\text{rad}}{\text{s}} \right]$$

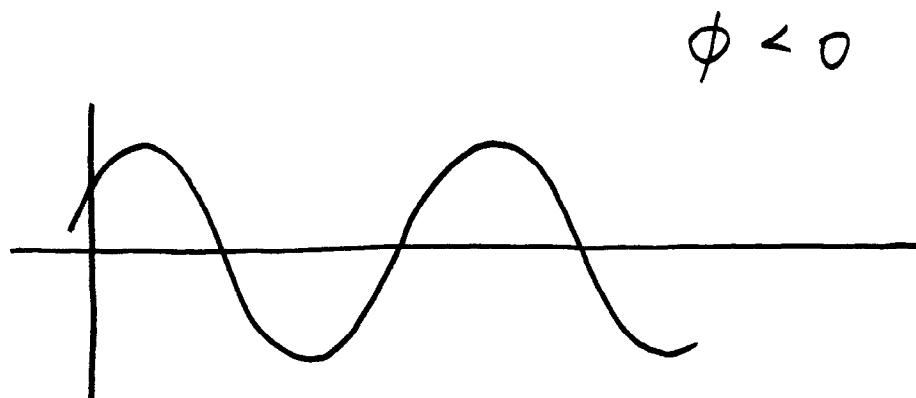
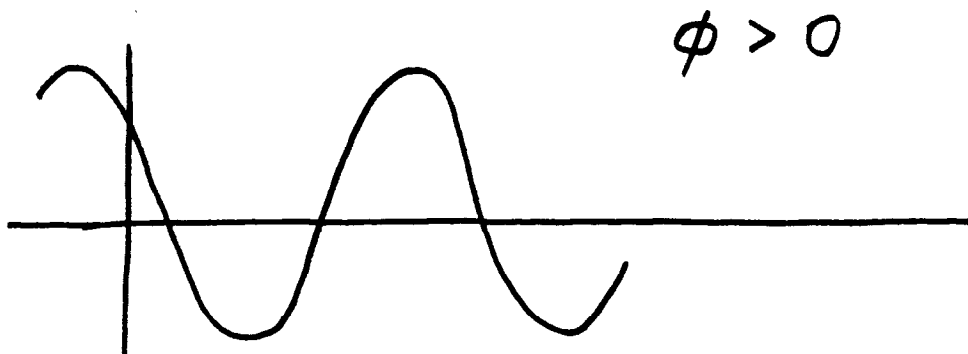
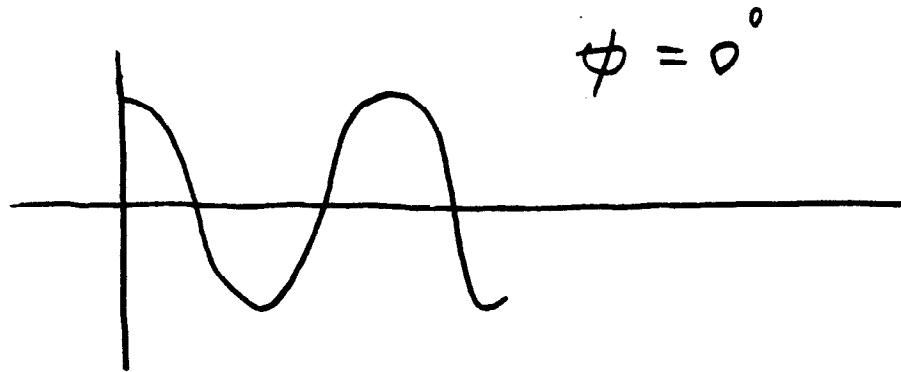
$\omega$  - angular freq.

$\phi$  - phase angle

$$v(t) = V_m \cos(\omega t + \phi)$$

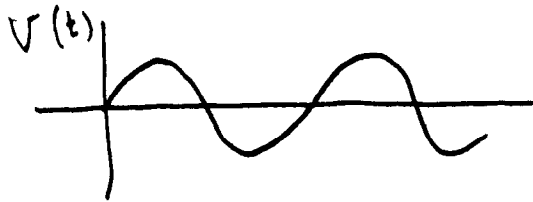


## Phase Shifts

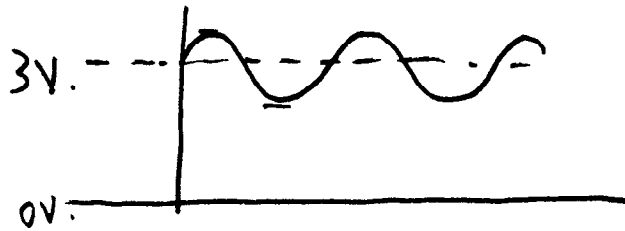


$$i(t) = I_m \cos(\omega t + \phi)$$

## Average Value



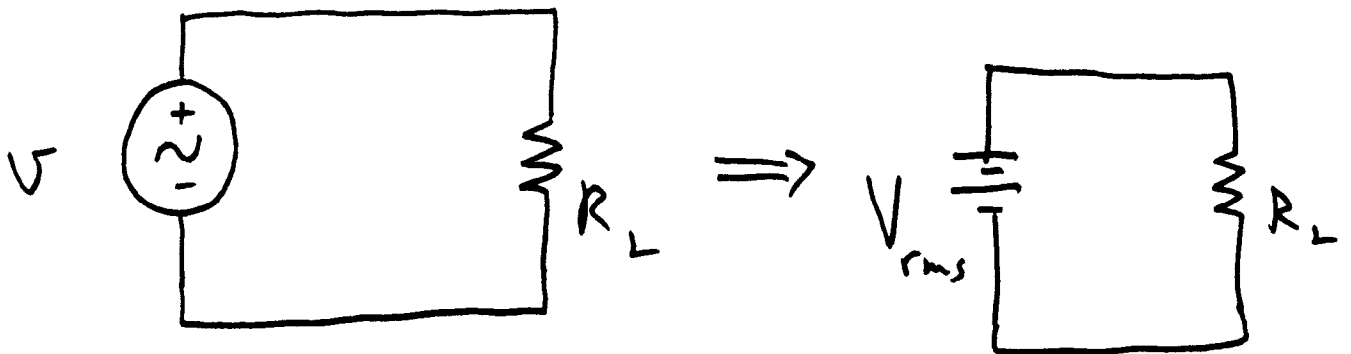
$$V_{av} = 0$$



$$v(t) = 3 + \frac{1}{2} \sin(\omega t)$$

$$\underline{V_{av} = 3}$$

## RMS Value, or Effective Value



$$V_{rms} = V_{eff}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2 dt}$$

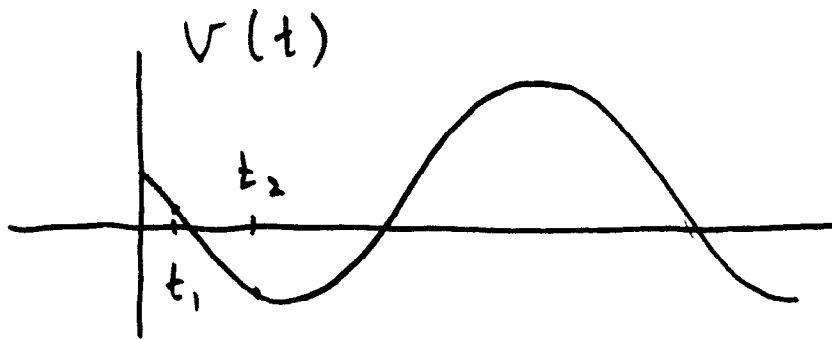
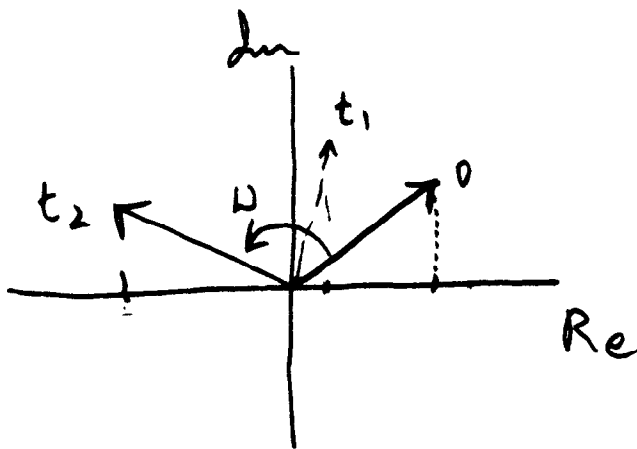
$$= \frac{1}{\sqrt{2}} V_m \quad \text{for } V_{av} = 0$$

## Phasors

L vector in complex plane

L fixed for analysis

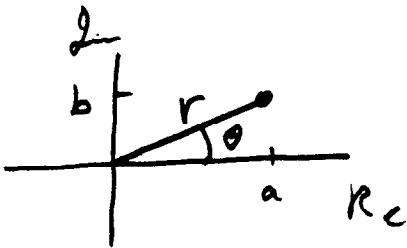
L rotates at  $\omega$



$$v(t) = V_m \cos(\omega t + \phi)$$

$$V = V_m \angle \phi$$

# Complex Algebra of Phasors 6-5

$$C = a + jb$$


$$= r \angle \theta$$

$$r = \sqrt{a^2 + b^2} \quad \theta = \tan^{-1} \left( \frac{b}{a} \right)$$

$C^*$  conjugate

$$C^* = a - jb$$

$$j = \sqrt{-1}$$
$$j^2 = -1$$

$$r = \sqrt{C^* C} = \sqrt{(a - jb)(a + jb)}$$
$$= \sqrt{a^2 + b^2}$$

$$C_1 \pm C_2 = (a_1 + a_2) + j(b_1 + b_2)$$

$$C_1 C_2 = (r_1 \angle \theta_1)(r_2 \angle \theta_2)$$
$$= (r_1 r_2) \angle \theta_1 + \theta_2$$

## 7. AC Circuits

7-1

$$\text{Impedance } Z = \frac{V}{I} = R + jX$$

$$(\text{analogous to } R = \frac{V}{I})$$

$$\left. \begin{array}{l} R = \text{resistance} \\ X = \text{reactance} \end{array} \right\} [\Omega]$$

$$R: Z_R = R \quad \text{pure real}$$

$$C: Z_C = \frac{1}{j\omega C} = jX_C \quad \text{pure imag}$$

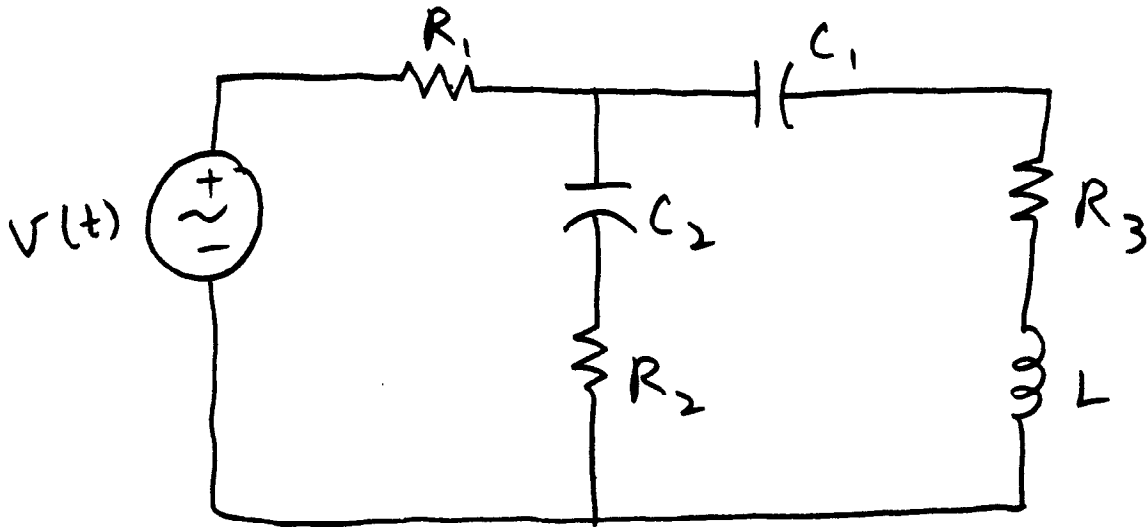
$$X_C = -\frac{1}{\omega C}$$

$$\frac{1}{j} = -j \quad (-j^2 = 1)$$

$$L: Z_L = j\omega L = jX_L; X_L = \omega L$$

7-2

Converting an AC circuit to its phasor equivalent



$$v(t) = 5 \cos(2000\pi t + 40^\circ)$$

$$V_m = 5 \text{ V.} \quad \omega = 2000\pi = 2\pi f$$

$$f = 1000 \text{ Hz}$$

$$\phi = 40^\circ$$

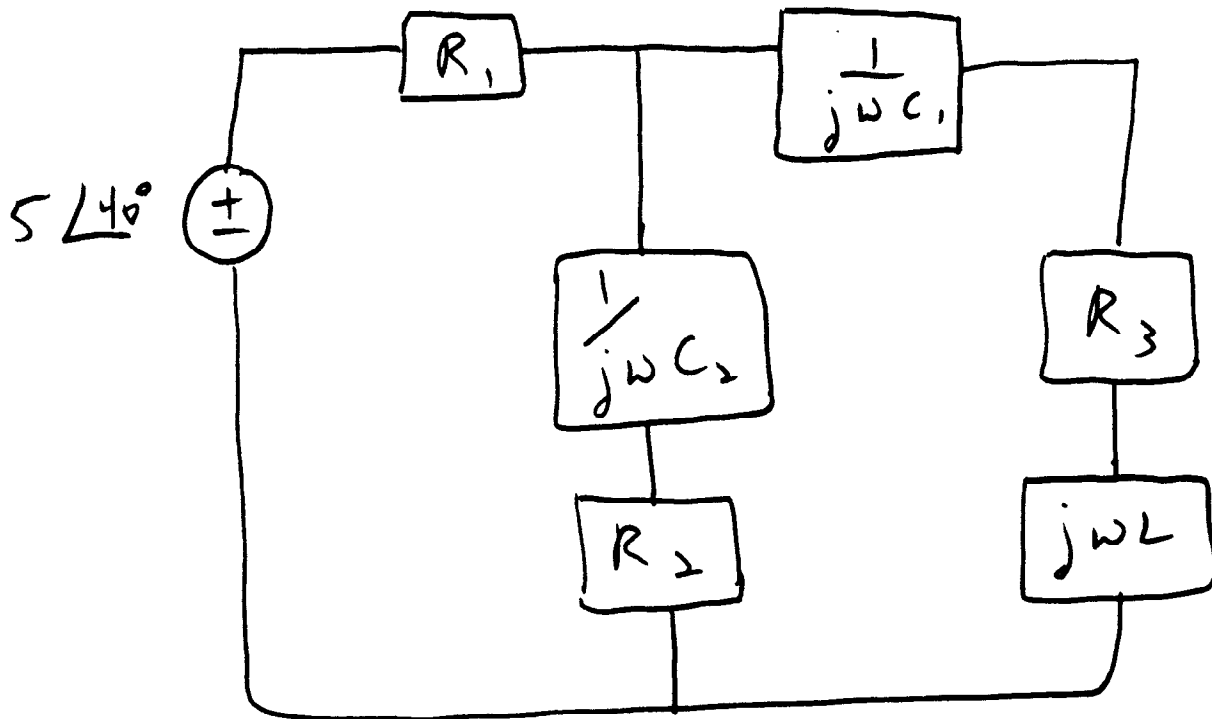
$$\mathbb{V} = 5 \angle 40^\circ$$

$$Z_{C1} = \frac{1}{j\omega C_1} \quad Z_{C2} = \frac{1}{j\omega C_2}$$

$$Z_L = j\omega L$$

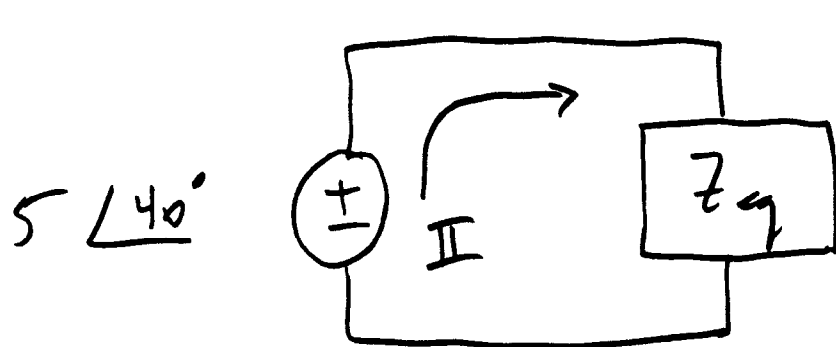
7-2.1

Phasor equivalent circuit



$$Z_{su} = \sum_{n=1}^N Z_n$$

$$Z_{pu} = \frac{1}{\sum_{n=1}^N \frac{1}{Z_n}}$$



$$V = Z II$$

$$II = \frac{5 \angle 40^\circ}{Z_{eq}}$$

# Circuit Solution Techniques 7-3

Same as for DC, with phasors for  $V$  and  $I$ , impedance for resistance.

Loop current

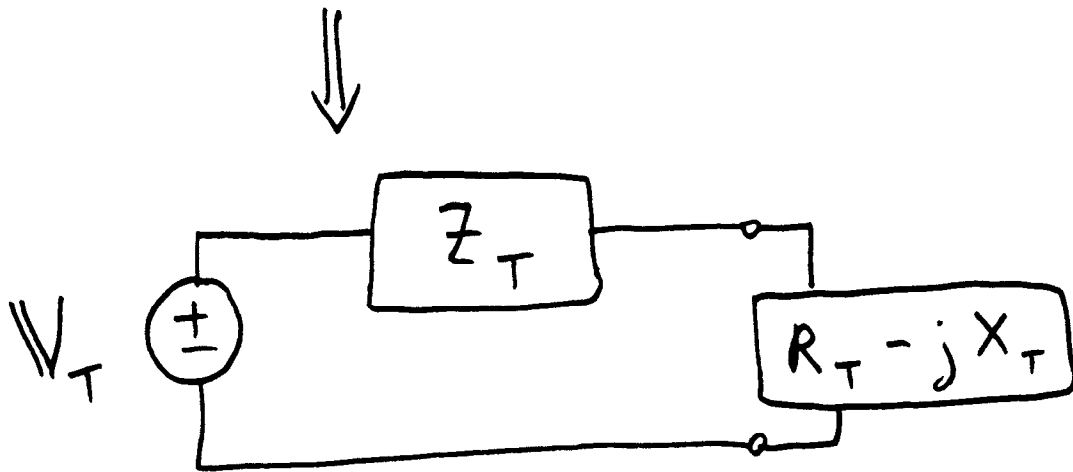
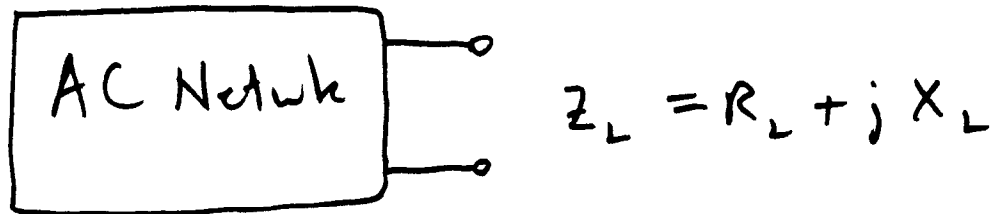
Node voltages

Thevenin / Norton

Superposition



# Maximum Power Transfer for AC circuit.



$$Z_T = R_T + jX_T$$

$$Z_L = R_T - jX_T$$

$$Z_{in} = 2R_T \quad (\text{imaginary part is cancelled})$$

$$Z_L = Z_T^*$$

# Complex Power

7-5

$$p(t) = v(t)i(t)$$

phasors

$$S = V_{\text{eff}} I_{\text{eff}}^*$$

$$= P + jQ$$

P power

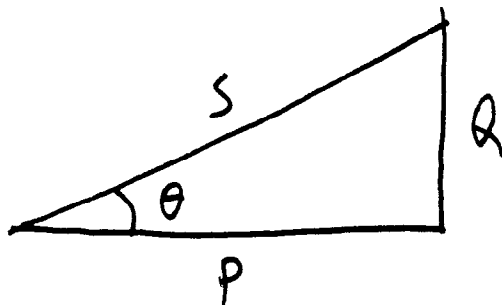
reactive power Q

Power Triangle, pf, rf

$$S = [V \cdot A]$$

$$P = [\text{Watts}]$$

$$Q = [\text{VAR}]$$



$$\cos \theta \equiv \text{pf}$$

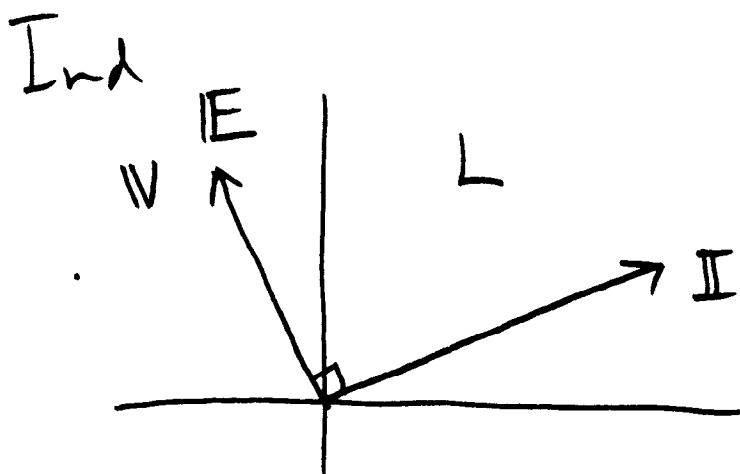
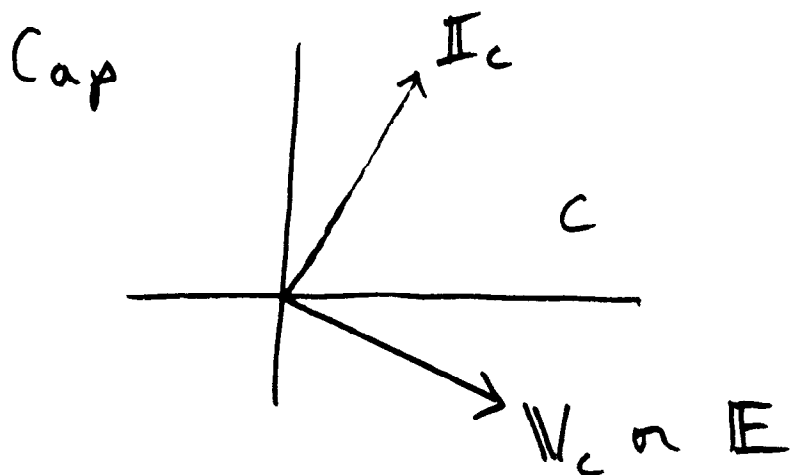
$$\sin \theta \equiv \text{rf}$$

pf - power factor

Leading and Lagging pf 7-6

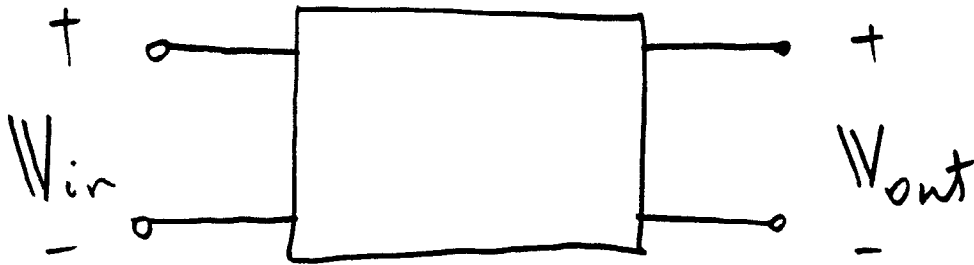
Capacitor:  $I$  leads  $V$  by  $90^\circ$   
Inductor:  $I$  lags  $V$  by  $90^\circ$

Phasor diagrams, ELI the ICEman



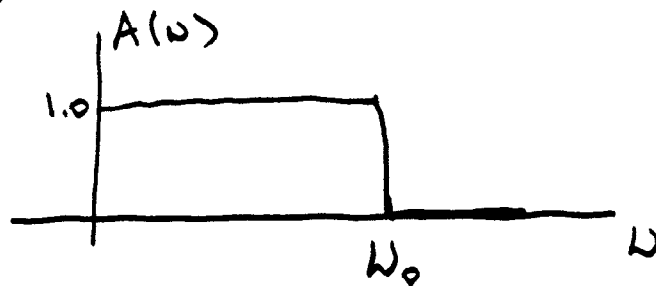
# Frequency Response

$V_{out}$  vs.  $V_{in}$  for various freq.

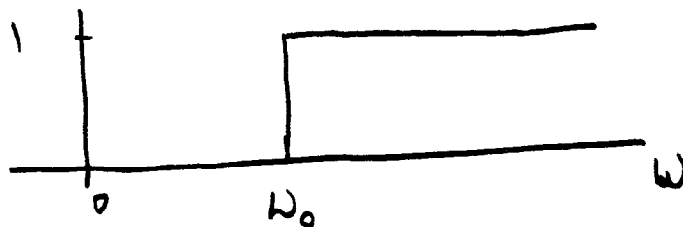


$$\frac{V_{out}}{V_{in}} \equiv A(\omega) \angle \phi(\omega)$$

Lowpass (Ideal)

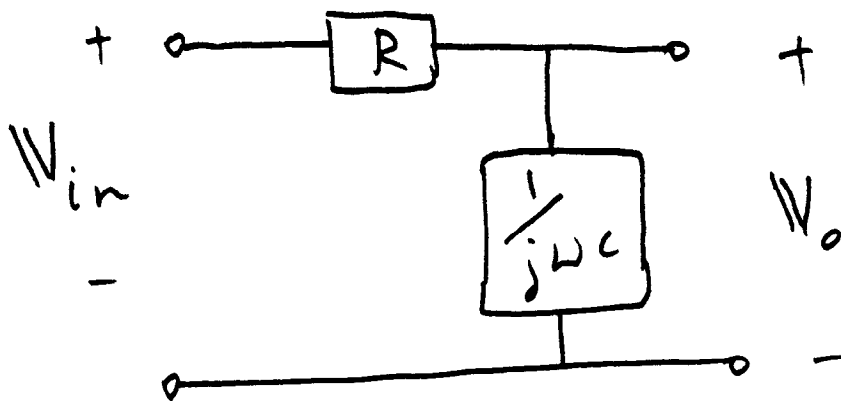
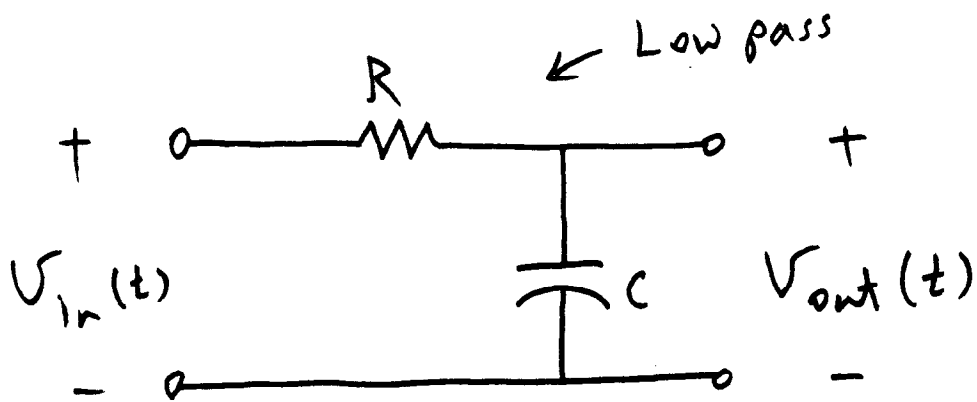


High pass (Ideal)



# RC Filters

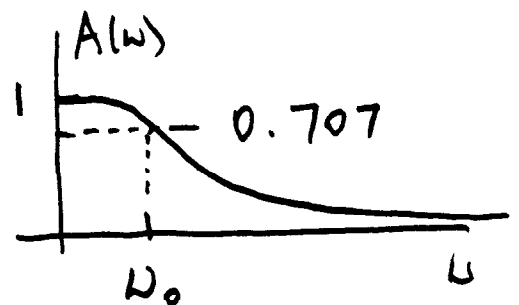
7-8



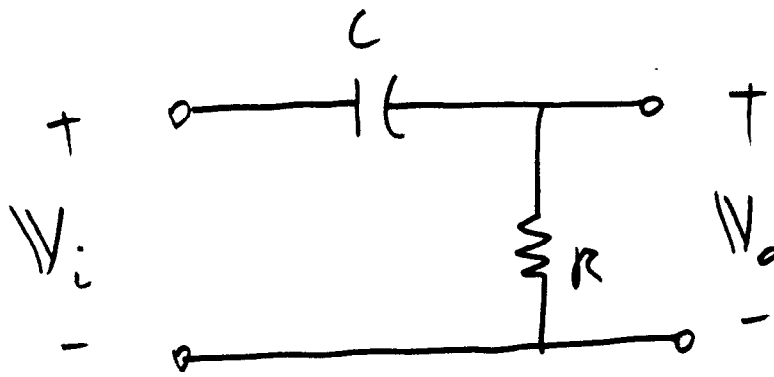
$$V_o = V_{in} \left( \frac{1/j\omega C}{1/j\omega C + R} \right)$$

$$\frac{V_o}{V_{in}} = \frac{1}{1 + j\omega RC} \quad \frac{1}{RC} \equiv \omega_0$$

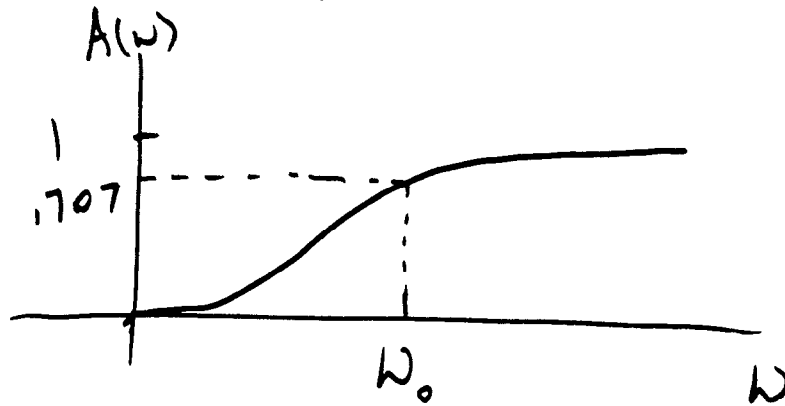
$$= \frac{1}{1 + j \frac{\omega}{\omega_0}}$$



7-8.1

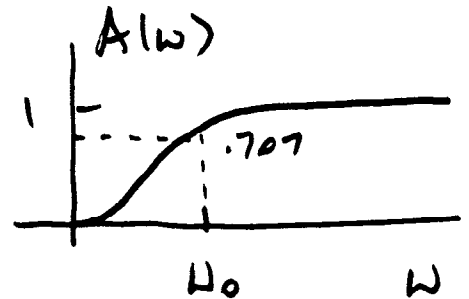
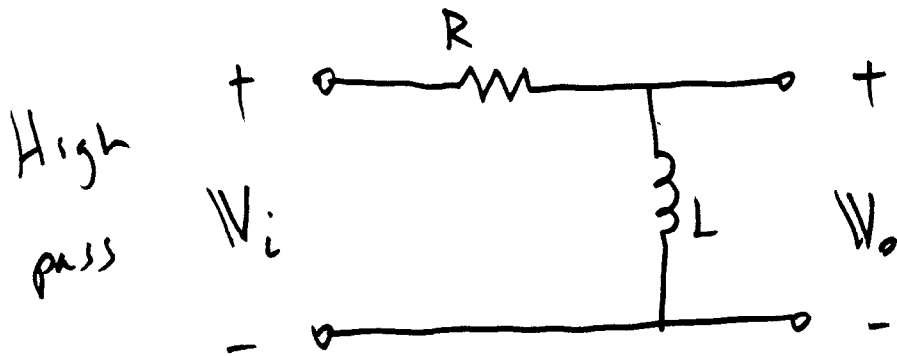
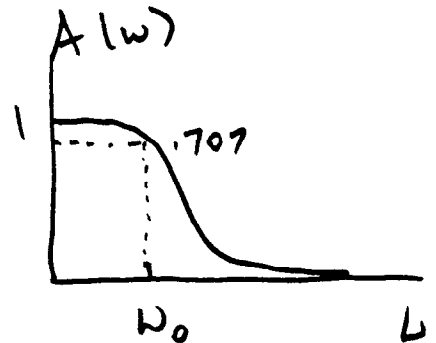
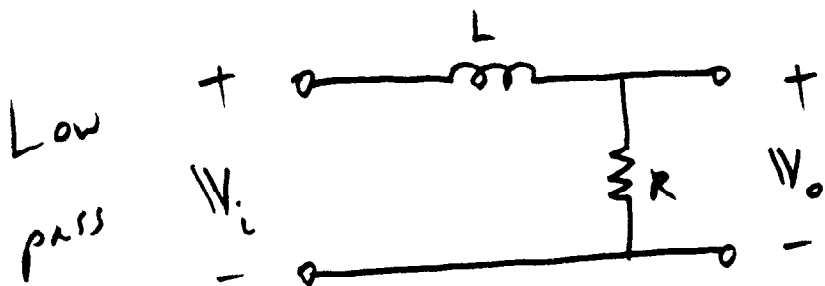


High pass



# RL Filters

7-9



$$Z_L = j\omega L$$

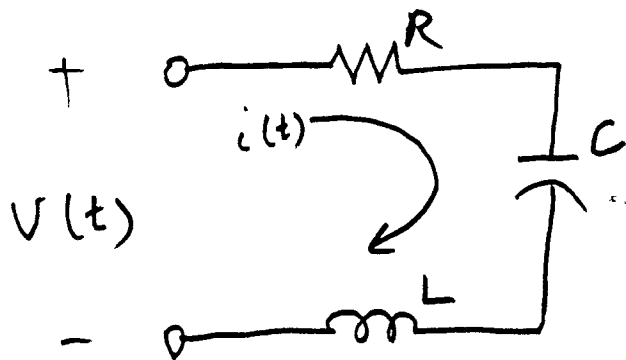
$$Z_L(\omega=0) = 0$$

$$Z_L(\omega \rightarrow \infty) \rightarrow \infty$$

Recall  $\tau = \frac{L}{R} \rightarrow \frac{R}{L} \equiv \omega_0$

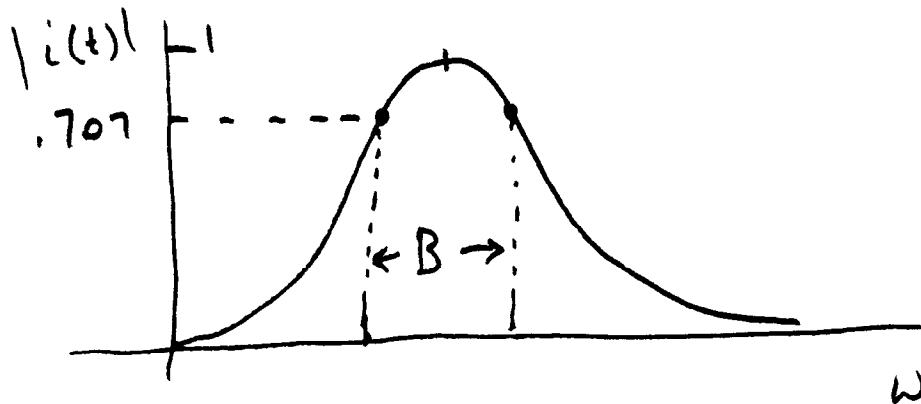
# RLC Filters and Resonance

7-10



$$Z_C = \frac{1}{j\omega C}$$

$$Z_L = j\omega L$$



$$Z_{eq} = R + j\omega L - j \frac{1}{\omega C}$$

$$= R + j \left( \omega L - \frac{1}{\omega C} \right)$$

When  $\omega_0 L = \frac{1}{\omega_0 C}$ ,  $i(t)$  max.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \left[ \frac{\text{rad}}{\text{s}} \right]$$

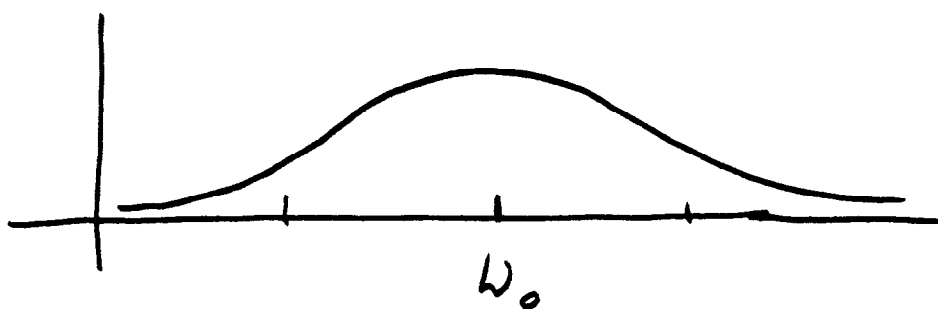
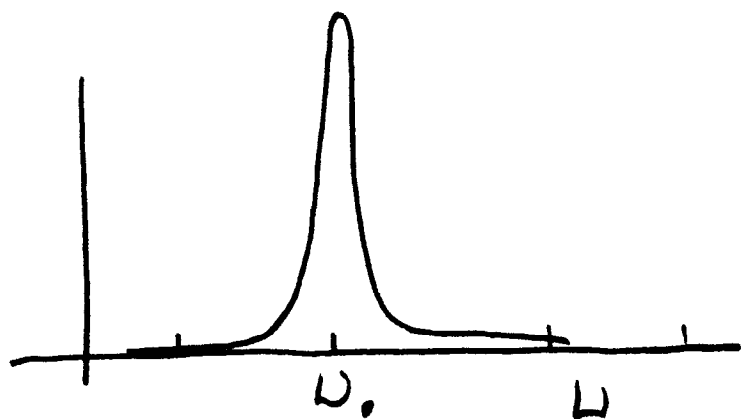


7-11

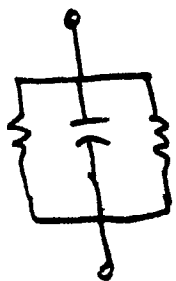
Bandwidth B

Resonant freq.  $\omega_0$

Quality factor  $Q = \frac{\omega_0}{B}$



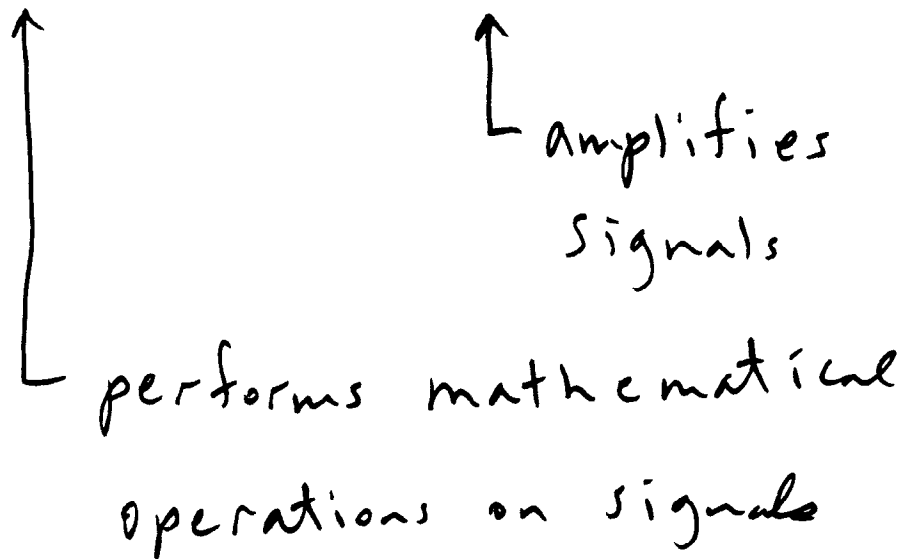
Series res.  $Q = \frac{\omega_0 L}{R}$



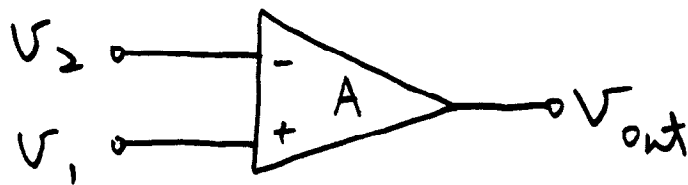
Parallel  $Q = \omega_0 RC$

## 8. Op Amps

### Operational Amplifier



### Circuit Symbol



$$V_{out} = A(V_1 - V_2)$$

## Ideal Op Amp \*

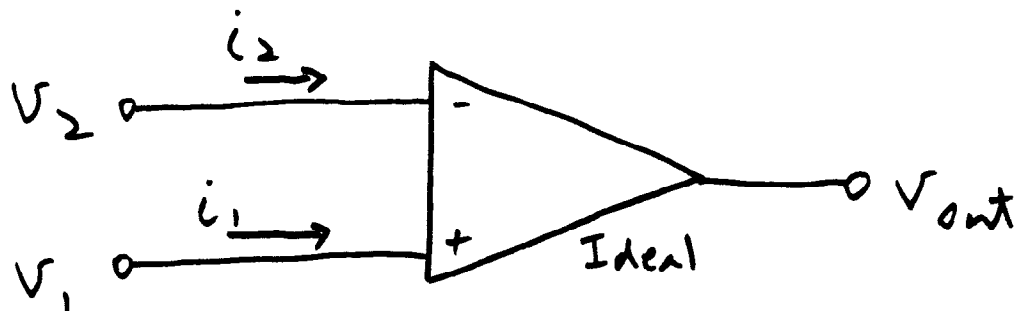
Let  $A \rightarrow \infty$ 

$$(V_1 - V_2) = \frac{V_{out}}{A} \xrightarrow{A \rightarrow \infty} 0$$

$$\text{So } \boxed{V_1 = V_2} *$$

Also (related limit)

$$\boxed{i_1 = 0, i_2 = 0} *$$



Relationships among inputs and output due to external resistors.

# Op Amp Circuits

Inverting Amp

Non-inverting Amp

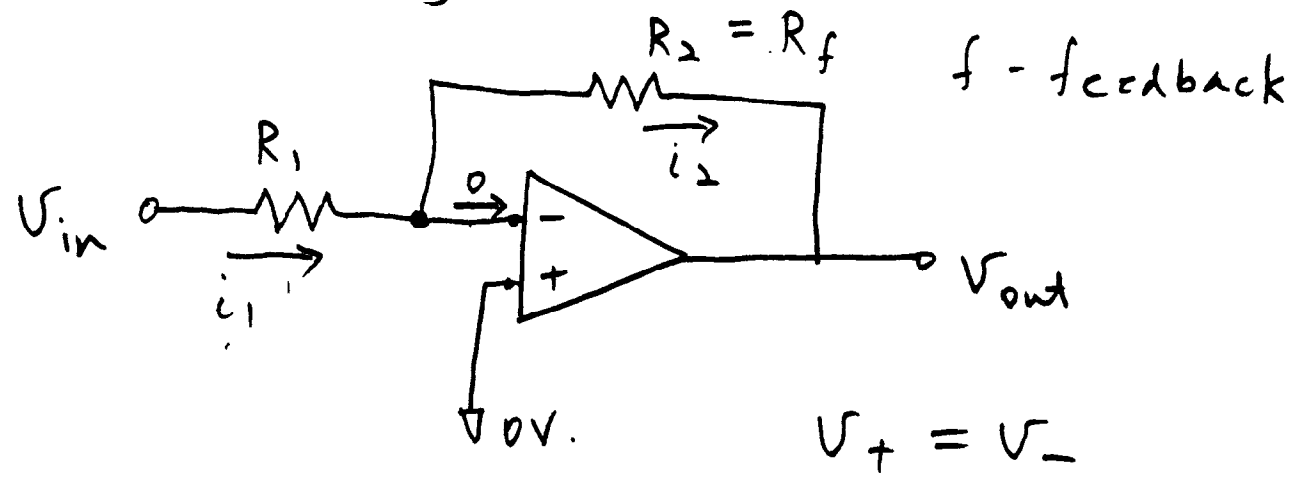
Summing Amp

Difference Amp

Integrator

Differentiator

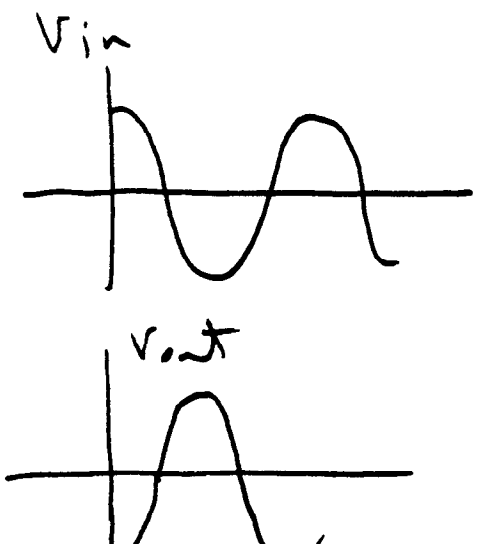
# Inverting Amp



KCL  $i_1 = 0 + i_2$

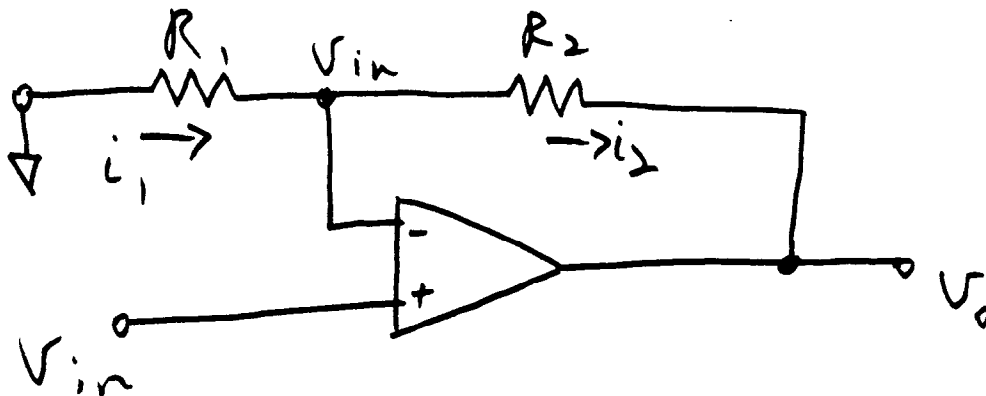
$$\frac{V_{in} - 0}{R_1} = \frac{0 - V_{out}}{R_2}$$

$$V_{out} = - \frac{R_2}{R_1} V_{in}$$



↑  
inv.

## Non-Inverting Amp

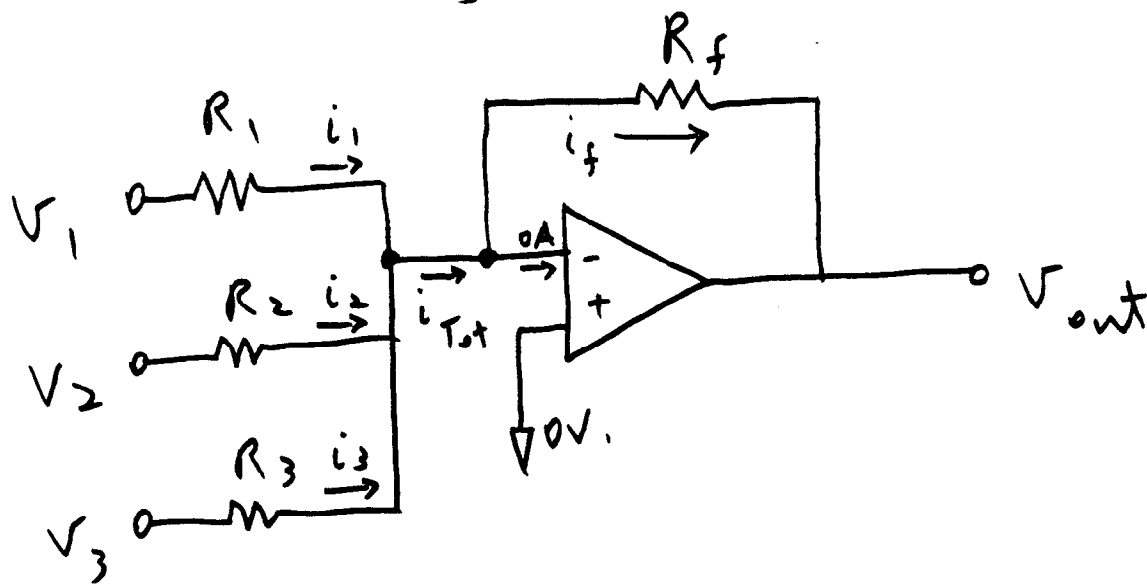


$$i_1 = i_2$$

$$\frac{0 - V_{in}}{R_1} = \frac{V_{in} - V_o}{R_2}$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

## Summing Amp



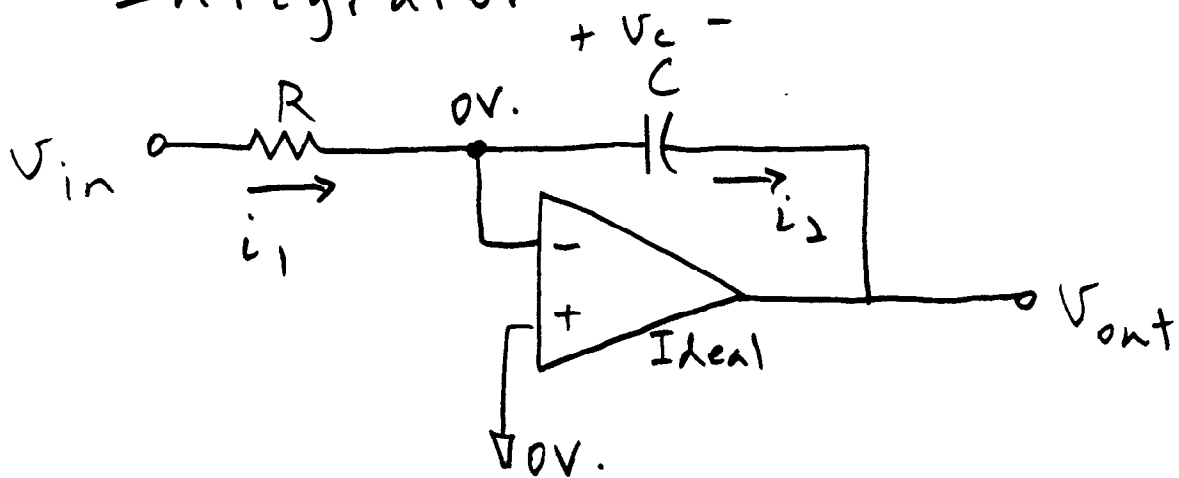
$$i_{Tot} = i_f$$

$$\frac{V_1 - 0}{R_1} + \frac{V_2 - 0}{R_2} + \frac{V_3 - 0}{R_3} = \frac{0 - V_{out}}{R_f}$$

$$V_{out} = - \left( \frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 \right)$$

- output is a weighted sum of the inputs.
- easily extended to more inputs

## Integrator



$$i_1 = i_2$$

$$\frac{V_{in} - 0}{R} = C \frac{d}{dt} V_c$$

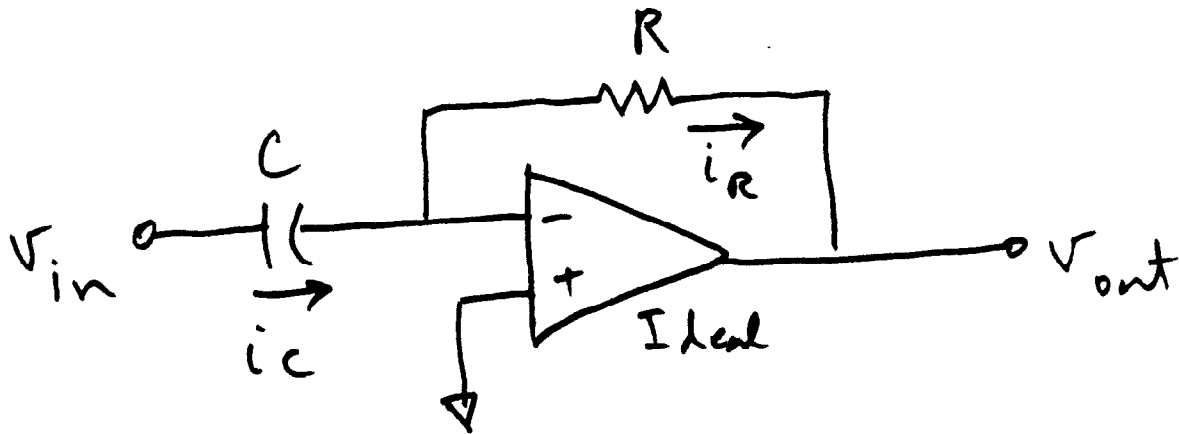
$$= C \frac{d}{dt} (0 - V_{out})$$

$$\frac{V_{in}}{RC} = - \frac{d}{dt} V_{out}$$

$$V_{out} = - \frac{1}{RC} \int V_{in} dt$$



## Differentiator



$$i_C = i_R$$

$$C \frac{d}{dt} (v_{in} - 0) = \frac{0 - v_{out}}{R}$$

$$v_{out} = -RC \frac{dv_{in}}{dt}$$

## References

- [1] NCEES FE Discipline Specific Reference Handbook
- [2] EIT Review Manual, by Michael R. Lindeburg, PE
- [3] Principles and Applications of Electrical Engineering, by Giorgio Rizzoni