Review for Fundamentals of Engineering Exam Computers and Numerical Methods

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OVERVIEW

I. Introduction
II. Computers
III. Numerical Methods

I. Introduction

A. FE Exam Organization

Session	**Topics**	**No./Total**
Morning		
All	Computers	7/120
Afternoon		
Chemical	Computers and Numerical Methods	3/60
Civil	C/NM	6/60
Industrial	Computer Computations and Modeling	3/60
Electrical	C/NM	3/60
General	Computers	3/60
Mechanical	C/NM	3/60

B. Sources for Review

1. RH: National Council of Examiners for Engineering and Surveying (NCEES) Reference Handbook, January 1996

2. NCEES: NCEES Sample Questions

3. ERM: EIT Review Manual, Michael Lindberg, Professional Publications Inc., 1997.

4. FE: Fundamentals of Engineering, Potter

II. Computers

A. Expectations (RH)

"Computer Knowledge
Examinees are expected to posses a level of computer expertise required to perform in a typical undergraduate environment. Thus only generic problems that do not require knowledge of a specific language or computer type will be required. Examinees are expected to be familiar with ***flow charts, pseudo code, and spread sheets*** *(Lotus, Quatro-Pro, Excel, etc.)"* (emphasis added)

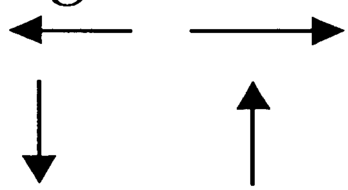
B. Major Topics

1. Flow Charts (NCEES, ERM))

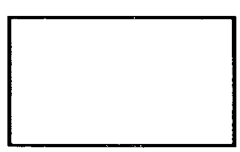
a) Symbols

(1) Terminal (beginning or end of program)

(2) logic flow

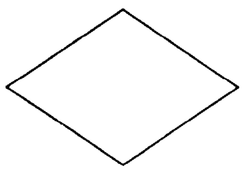


(3) process (calculations or data manipulation)

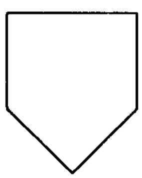


(4) input/output

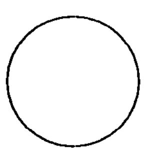
(5) decision
(comparison, question, decision)



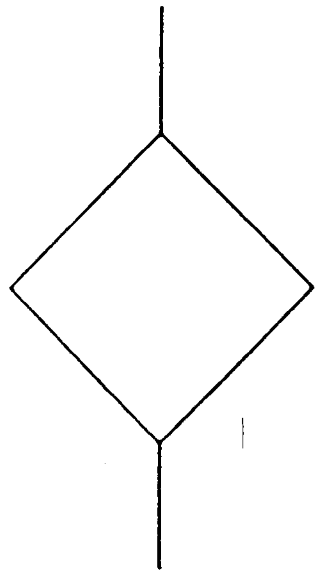
(6) off page connector



(7) on-page connector



11. What operation is typically represented by the following program flow-chart symbol?



(A) input-output
(B) processing
(C) storage
(D) branching

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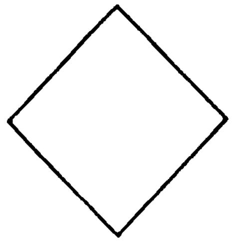
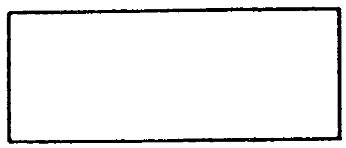
Solution:

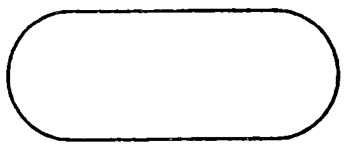
Branching, comparison, and decision operations are typically represented by the diamond symbol.

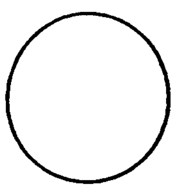
Answer is D.

16. Which of the following program flowchart symbols is typically used to indicate the end of a process or program?

(A) 

(B) 

(C) 

(D) 

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Solution 16:

Termination (end of the program) is commonly represented by the symbol in choice (C).

2. Pseudo Code

a) assignment statements
(e.g. X = 3.4)

b) INPUT
(e.g. INPUT X, N)

c) FOR/NEXT loop
(e.g. FOR I = 1 TO N; NEXT I)

d) DOWHILE/ENDDO
(e.g. DOWHILE A < 3.4 …
ENDDO)

e) DOUNTIL/ENDDO
(e.g. DOUNTIL Y > 5.0…
ENDO)

f) IF/ELSE/ENDIF
(e.g. IF I = 2 THEN …
ELSE … ENDIF)

56. The program segment

```
INPUT Z, N
S = 1
T = 1
FOR K = 1 TO N
T = T*Z/K
S = S + T
NEXT K
```

calculates the sum

(A) $S = 1 + ZT + 2\,ZT + 3\,ZT + \ldots + nZT$
(B) $S = 1 + ZT + (^1/_2)\,ZT + (^1/_3)\,ZT + \ldots + (^1/_n)\,ZT$
(C) $S = 1 + Z/1 + Z^2/2 + Z^3/3 + \ldots Z^n/n$
(D) $S = 1 + Z/1! + Z^2/2! + Z^3/3! + \ldots + Z^n/n!$

Solution

K	1	2	3
T	$\frac{(1)Z}{(1)}$	$\frac{Z \cdot Z}{2} = \frac{Z^2}{2}$	$\frac{Z^2}{2}\frac{Z}{3} = \frac{Z^3}{(2)(3)}$
S	$1 + Z$	$+ \frac{Z^2}{2}$	$+ \frac{Z^3}{(2)(3)}$

→ D

23. How many times will the second line be executed

```
              M = 42
LOOPSTART     M = M - 1
              P = INTEGER PART OF (M/2)
              IF P > 15, THEN GO TO
                  LOOPSTART, OTHERWISE
                  GO TO END
END           PRINT "DONE"
```

(A) 8
(B) 9
(C) 10
(D) 11

Solution 23:

The values of the variables for each iteration are

iteration	M	P
1	41	20
2	40	20
3	39	19
4	38	19
5	37	18
6	36	18
7	35	17
8	34	17
9	33	16
10	32	16
11	31	15

3. Spreadsheets

a) organization

(1) cells

(2) rows/columns

b) cell contents

(1) numbers

(2) labels

(3) expressions

(a) operators

(e.g. +, -, *, /, ^)

(b) functions

(e.g. exp(), log(), sin(), abs())

c) addresses
   (1) relative address (when copied refers to cell in same relative position)
   (2) absolute address (when copied refers to same cell)

55. In a spreadsheet, the number in cell A4 is set to 6. Then A5 is set to A4 + A4, where $ indicators absolute cell address. This formula is copied into cells A6 and A7. The number shown in cell A7 is most nearly:

(A) 12
(B) 24
(C) 36
(D) 216

	A	
4	6	(6)
5	=A4 + A4	(12)
6	=A5 + A4	(18)
7	=A6 + A4	(24)

ANSWER: (B) 24

17. The cells in a computer spreadsheet program are as shown.

	A	B	C	D
1	4	−1	3	0
2	3	A3+C1	−3	6
3	0.5	m	33	2
4	Smith	1	B2*B3	C4

Instructions in macro-commands are scanned from left to right. What is the value of n in the following macro-commands?

$$m = 5$$

$$p = m * 2 + 6$$

$$n = D4 - 3 * p * *0.5$$

(A) 4
(B) 5.5
(C) 10.5
(D) 1041

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lution 17:

raluate the macro. Use the following typical prece-
nce rules:

- exponentiation before multiplication and division
- multiplication and division before addition and subtraction
- operations inside parentheses before operations outside

$$\begin{aligned}
m &= 5 \\
p &= m * 2 + 6 = (5 * 2) + 6 = 16 \\
n &= D4 - 3 * p * *0.5 \\
&= C4 - 3 * p * *0.5 \\
&= B2 * B3 - 3 * p * *0.5 \\
&= (A3 + C1) * m - 3 * p * *0.5 \\
&= ((0.5 + 3) * 5) - (3) * (16) * *(0.5) \\
&= ((3.5) * 5) - (3) * (16) * *(0.5) \\
&= ((3.5) * 5) - 12 \\
&= 17.5 - 12 \\
&= 5.5
\end{aligned}$$

swer is B.

3-B-3

III.Numerical Methods

A. Root of Equations

1. Newton

(RH, Mathematics, p 18):

$$a_i^{j+1} = a_i^j - \left.\frac{P(x)}{\partial P(x)/\partial x}\right|_{x=a_i^j}$$

$$P(x) = x^n + \alpha_1 x^{n-1} + \alpha_2 x^{n-2} + \ldots + \alpha_{n-1}$$

Notes: 1) The method is shown for polynomials, but it is also applicable to other functions

2) There is confusion in the nomenclature used. "α" is used a as polynomial coefficient and "a" is used as an estimate of the root

3) The final term in the definition of the polynomial should be "α_n" rather than "α_{n-1}"

Find the root of

$P(x) = x^3 - 11x^2 + 4x + 60 = 0$

on the interval 5-20

Solution

$$\frac{dP}{dx} = 3x^2 - 22x + 4$$

let $a_0 = \frac{5+20}{2} = 12.5$

$$a_1 = a_0 - \frac{P(a_0)}{\left(\frac{dP}{dx}\right)_{x=a_0}}$$

$$= 12.5 - \frac{(12.5)^3 - 11(12.5)^2 + 4(12.5) + 60}{3(12.5)^2 - 22(12.5) + 4}$$

$$= 10.8$$

$$a_2 = 10.8 - \frac{(10.8)^3 - 11(10.8)^2 + 4(10.8) + 60}{3(10.8)^2 - 22(10.8) + 4}$$

$$= 10.1$$

$$a_3 = 10.1 - \frac{(10.1)^3 - 11(10.1)^2 + 4(10.1) + 60}{3(10.1)^2 - 22(10.1) + 4}$$

$a_4 = 10.0$

2. Bisection (ERM)

a) start with interval containing root: $a_l < a_i < a_u$

b) bisect interval: $a_b = (a_l + a_u)/2$

c) check sub- intervals

(1) if $P(a_b)^* P(a_l) < 0$, then

$a_l < a_i < a_b$

(2) if $P(a_b)^* P(a_u) < 0$, then

$a_b < a_i < a_u$

d) retain sub-interval with a_i

e) redefine limits

$a_u = a_b$ or $a_l = a_b$

f) continue until change in estimate of root (a_b) is sufficiently small.

B. Minimization with Newton's Method

(RH, Mathematics, p 18)

find a vector **x** (x_1, x_2, ...) that will minimize the scalar function $h(\mathbf{x}) = h(x_1, x_2, \ldots x_n)$

$$\mathbf{x}_{K+1} = \mathbf{x}_K - \left(\left. \frac{\partial^2 h}{\partial x^2} \right|_{\mathbf{x}=\mathbf{x}_k} \right)^{-1} \left. \frac{\partial h}{\partial x} \right|_{\mathbf{x}=\mathbf{x}_K}$$

where:

$$\frac{\partial h}{\partial x} = \begin{bmatrix} \dfrac{\partial h}{\partial x_1} \\ \dfrac{\partial h}{\partial x_2} \\ \cdots \\ \cdots \\ \dfrac{\partial h}{\partial x_n} \end{bmatrix} \qquad \frac{\partial^2 h}{\partial x^2} = \begin{bmatrix} \dfrac{\partial^2 h}{\partial x_1^2} & \dfrac{\partial^2 h}{\partial x_1 \partial x_2} & \cdots & \cdots & \cdots & \dfrac{\partial^2 h}{\partial x_1 \partial x_n} \\ \dfrac{\partial^2 h}{\partial x_1 \partial x_2} & \dfrac{\partial^2 h}{\partial x_2^2} & \cdots & \cdots & \cdots & \dfrac{\partial^2 h}{\partial x_2 \partial x_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial^2 h}{\partial x_1 \partial x_n} & \dfrac{\partial^2 h}{\partial x_2 \partial x_n} & \cdots & \cdots & \cdots & \dfrac{\partial^2 h}{\partial x_n^2} \end{bmatrix}$$

Notes:

1) Method is applicable to solving sets of non-linear equations ($f_i(\boldsymbol{x})=0$), where $\partial h(\boldsymbol{x})/\partial x_i = f_i(\boldsymbol{x})$.

2) Method is applicable to non-linear regressions, where $h(\boldsymbol{x})$ is the sum of squared errors and $\boldsymbol{x}$ is the vector of unknown coefficients.

C. Integration

(RH, Mathematics, p 18)

$\Delta x=(b-a)/n$

n= number of segments

a,b are limits of integration

1. Euler's or Forward Rectangular Rule

$$\int_a^b f(x)dx \approx \Delta x \sum_{k=0}^{n-1} f(a+k\Delta x)$$

Find $\int_0^8 f(x)\,dx$ by Forward Rectangle

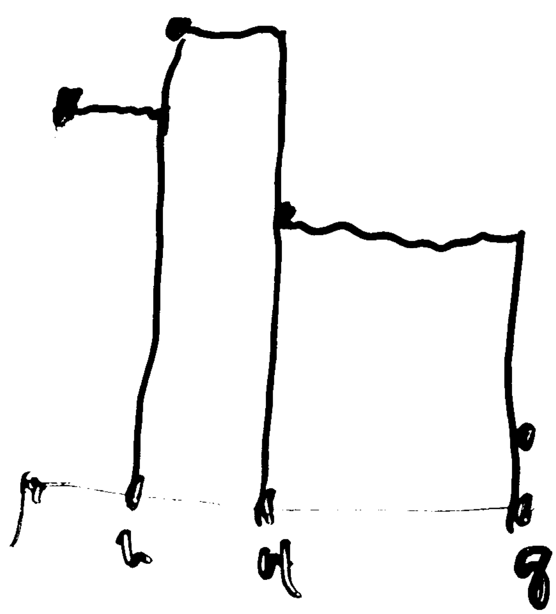
X	f(x)
0	10
2	12
4	7
8	2

$$\int_a^b f(x)\,dx = \Delta x \sum_{k=0}^{N-1} f(a+k\Delta x)$$

$$= \Delta x_1 \sum f(a+k\Delta x) + \Delta x_2 \sum f(a+k\Delta x)$$

$$= 2(10+12) + 4(7)$$

$$= 72.0$$



2. Trapezoidal Rule

a) n=1

$$\int_a^b f(x)dx \approx \Delta x\left[\frac{f(a)+f(b)}{2}\right]$$

b) n>1

$$\int_a^b f(x)dx \approx$$

$$\frac{\Delta x}{2}\left[f(a)+2\sum_{k=1}^{n-1} f(a+k\Delta x)+f(b)\right]$$

Find $\int_0^8 f(x)\,dx$ by trapezoidal

x	$f(x)$
0	10
2	12
4	7
8	2

$$\int_a^b f(x)\,dx = \frac{\Delta x}{2}\left[f(a) + 2\sum_{k=1}^{n-1} f(a + k\Delta x) + f(b)\right]$$

apply to each region with constant Δx

$$\int_0^8 = \frac{2}{2}\left[10 + 2(12) + 7\right] + \frac{4}{2}\left[7 + 2\right]$$

$$= 49.0$$

3. Simpson's Rule/Parabolic Rule

a) n=2 (two segments)

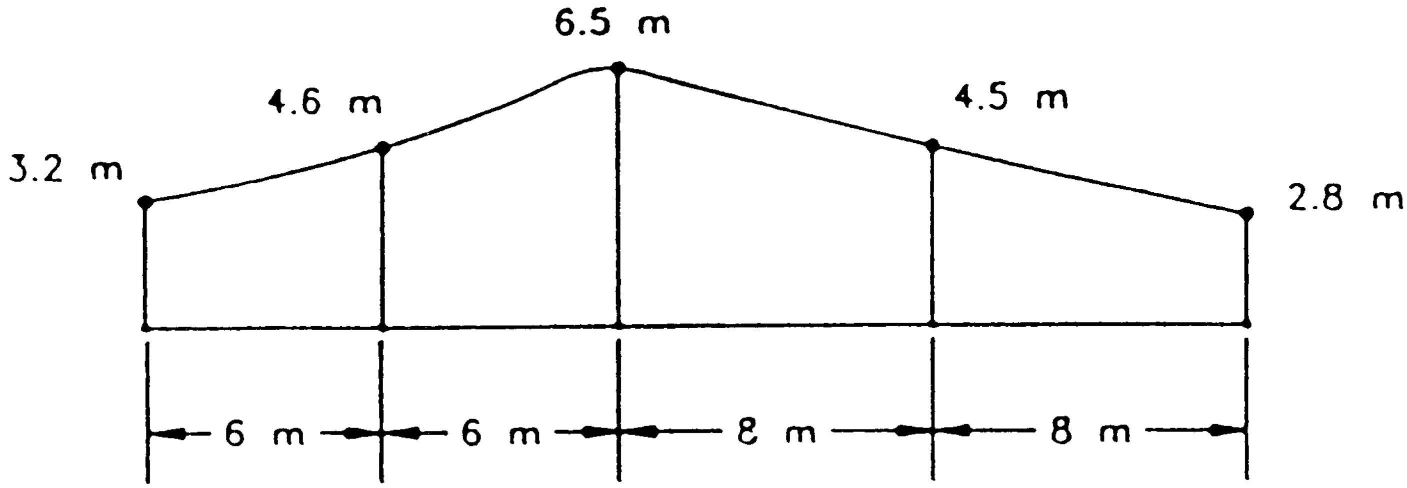
$$\int_a^b f(x)dx \approx \left(\frac{b-a}{6}\right)\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

$$=\frac{\Delta x}{3}\left[f(a)+4f(a+\Delta x)+f(b)\right]$$

b) n>= 4, and be an even integer

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}\left[\begin{array}{l} f(a)+2\sum_{k=2,4,6,\ldots}^{n-2} f(a+k\Delta x) \\ +4\sum_{k=1,3,5,\ldots}^{n-1} f(a+k\Delta x)+f(b) \end{array}\right]$$

28. The following diagram is an offset measurements for an irregular tract.



Using Simpson's 1/3 rule, the area of the tract (m^2) is most nearly:

(A) 121.0
(B) 129.0
(C) 129.5
(D) 129.9

To each segment with constant Δx

$\Delta x = 6\,m$

$$\int_0^{12} f(x)\,dx \approx \left(\frac{12-0}{6}\right)\left[3.2 + 4(4.6) + 6.5\right]$$

$$= 56.2$$

$$\int_{12}^{28} f(x)\,dx = \left(\frac{28-12}{6}\right)\left[6.5 + 4(4.5) + 2.8\right]$$

$$= 72.8$$

Total Area $= 56.2 + 72.8 = 129.0$

Answer: (B) 129.0

D. Ordinary Differential Equation

1. Euler

(RH, Mathematics, p 18)

to find $y(t)$ for $t = a$ to $t = b$, given :

$$dy/dt = f(y,t) \text{ and } y(a) = y_a$$

use sequentially :

$$y[(k+1)\Delta t] \cong y(k\Delta t) + \Delta t \; f[y(k\Delta t), k\Delta t]$$

until $b = (k+1)\Delta t$

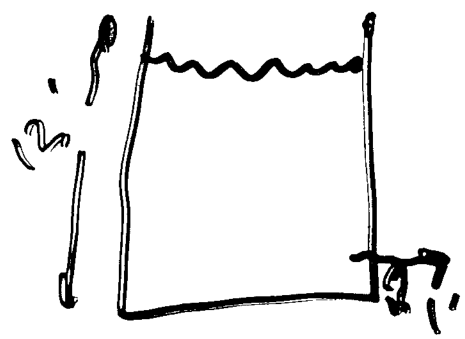
29. A chemical storage tank has a constant cross-sectional area of 90.0 square feet and a maximum liquid depth of 12.0 feet. Liquid is withdrawn from the tank through a pipe which is located 1.0 feet above the bottom of the tank. The flow rate is regulated by a valve as follows:

$$Q = 0.02\,H^{1/2}$$

in which Q = Flow Rate (CFS), and
H = liquid height above valve (ft)

There is no input of chemical to the tank. Using a centered difference approximation, the length of time (minutes) required to lower the liquid level in the tank from 11.0 feet to 9.0 feet is most nearly:

(A) 45.2
(B) 47.4
(C) 50.0
(D) 106.1



A mass balance with constant density (flow balance) is

$$\frac{dV}{dt} = Q_{in} - Q_{out}$$

$$V = AH$$

$$Q_{out} = 0.02\, H^{1/2}$$

$$Q_{in} = 0$$

$$\frac{dH}{dt} = -\frac{0.02\, H^{1/2}}{A}$$

$$\frac{H_2 - H_1}{\Delta t} = \frac{-0.02 \left(\frac{H_1 + H_2}{2}\right)^{1/2}}{A}$$

$$\Delta t = \frac{-(H_2 - H_1)\, A}{(0.02) \left(\frac{H_1 + H_2}{2}\right)^{1/2}}$$

$$= -\frac{(9 - 11\ ft)(90\ ft^2)}{(0.02\ ft^3/s \cdot ft^{1/2}) \left(\frac{9+11}{2}\right)^{1/2} ft^{1/2}}$$

$$= 2{,}846\ s = 47.4\ min$$

Answer: (B) 47.4

2. Application to nth order ODE

(mentioned in RH, Mathematics, p 18)

a) for second-order

solve: $$\frac{\partial^2 y}{\partial t^2} + f(x,y)\frac{\partial y}{\partial t} + g(x,y) = 0$$

given: $$y(0) = y_0, \quad \left(\frac{\partial y}{\partial t}\right)_{t=0} = yprime_0$$

define: $$z = \frac{\partial y}{\partial t}$$ then: $$\frac{\partial z}{\partial t} = \frac{\partial^2 y}{\partial t^2}$$

substituting: $$\frac{\partial z}{\partial t} + f(x,y)z + g(x,y) = 0$$

$$z(0) = yprime_0$$

$$\frac{\partial y}{\partial t} = z$$

$$y(0) = y_0$$

b) for higher order, repeat

(1 n^{th} order ODE $\Rightarrow$ n 1^{st} order ODE

E. Linear Least Squares Regression

(RH: Industrial Engineering, p 100)

$$y = \hat{a} + \hat{b}x$$

$$\text{where}: \hat{a} = \overline{y} - \hat{b}\overline{x}$$

$$\hat{b} = S_{xy} / S_{xx}$$

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - (1/n)\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)$$

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - (1/n)\left(\sum_{i=1}^{n} x_i\right)^2$$

$$\overline{y} = (1/n)\left(\sum_{i=1}^{n} y_i\right)$$

$$\overline{\mathrm{x}} = (1/n)\left(\sum_{i=1}^{n} x_i\right)$$

Linear Regression Example

Given following data for population in a town, estimate by linear regression the expected population in the year 2000.

Year	**Population**
1960	23,211
1970	29,985
1980	36,214
1990	43,591

x	y	xy	x^2
1960	23,211	45,493,560	3,841,600
1970	29,985	59,070,450	3,880,900
1980	36,214	71,703,720	3,920,400
1990	43,591	86,746,090	3,960,100
7900	133,001	263,013,820	15,603,000

Sxy=	= 263,013,820-(1/4)*(7900)*133,001)	=336,845
Sxx=	=15,603,000-(1/4)*(7900)^2	=500
y_bar=	=(1/4)*(133,001)	=33,250.25
x_bar=	=(1/4)*(7900)	=1975
b_hat=	=336,845/500	=673.69
a_hat=	=33,250.25-(673.69)*(1975)	=-1,297,288

at x=2000

y=-1,297,288 + (673.69)*(2000) =50,092